Investment and information acquisition*

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Abstract

We study the interaction between productive investment and persuasion activities in a principal-agent setting with strategic disclosure. In an attempt to persuade the principal, the agent diverts substantial resources from productive activities to information acquisition for persuasion, even though productive activities are more efficient and raise the chances of success in persuasion. The equilibrium outcomes of simultaneous and sequential allocation procedures are the same, because the value of learning and experimentation through information acquisition is dominated by the value of productive investment. We show that an increase in cost of an investment project leads to a lower productive investment.

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Information is a valuable good that fosters efficient allocation of limited resources. However, gathering information is not costless. Rather, it requires expending resources taken away from other uses. If the party that gathers information cannot fully appropriate the returns to information, then information can be undersupplied. At the same time, information is often used as a strategic instrument to influence and persuade decision makers, which creates additional stimuli to produce information. It is therefore natural to inquire whether in the presence of persuasion motives, information will be produced efficiently, overproduced or remain undersupplied.[1]

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[1] This question is relevant given the importance of persuasion activities in modern economies. Donald McCloskey and Arjo Klamer (1995) show that a substantial fraction of the US GDP is spent on persuasion activities. The more recent study of Gerry Antioch et al. (2013) largely confirms these findings.

This paper explores this issue by studying a novel tradeoff in resource allocation between productive investment and acquiring information for persuasion purposes. To this end we construct a simple principal-agent model in which an agent (he) wants to persuade a principal (she) to approve a project with an uncertain return. The agent allocates a fixed budget between *productive investment* that improves the project return stochastically, and *information acquisition* that generates verifiable signals about the project return. The principal observes neither the budget allocation nor the realization of the signals. The agent chooses which signals to disclose to the principal, and the principal decides whether to approve or reject the project.

To keep the model tractable, we consider a binary signal structure. Each signal is either a "success" or a "failure" where a success signals a higher project return, and a failure signals the opposite.[2] In this setup, the value of a signal comes entirely from its persuasion effect. This allows us to identify and highlight the persuasion motive in information acquisition without confounding it with other motives.

Our first result shows that in the setting with static budget allocation, in which the budget allocation decision is made once and for all, the agent invests substantial resources in information acquisition. This stands in contrast with the first-best outcome which prescribes spending the entire budget on productive investment. Intuitively, the agent faces he following tradeoff in allocating resources between investment and information acquisition. As productive investment improves the distribution of the project return, it also increases the probability that any given signal is a success, which incentivizes the agent to invest. In contrast, the distribution of the project return is not affected by information acquisition. However, acquiring more signals increases the chances that the agent obtains a sufficient number of successes to persuade the principal to implement the project. This motivates the agent to shift resources towards information acquisition.

Our central result shows that as the project cost increases, the agent shifts more resources into information acquisition at the expense of the productive investment. To understand why this is so, consider what happens as the cost of the project increases. In response, the principal has to raise her requirements for project approval: she now needs either more successful signals or a higher productive investment or both. But the only instrument at her disposal is a number of successful signals required for the project approval, and in deciding how to change it the principal has to consider the agent's equilibrium response. If the principal raises the signal threshold, the agent shifts resources towards information acquisition, away from the productive investment. But since lower productive investment decreases the chances that any given signal is a success, the rate at which the agent reduces productive investment after an increase in the signal threshold is less than 1-to-1. This turns out to be both a blessing and a curse. It is a blessing because productive investment does not fall too much in response to a higher

[2] There are many contexts where information comes in binary form. For example, the technology can either work or not, a certain task can be completed or left unfinished or a test can be failed or passed. Often, it is too costly or infeasible to observe or assess the "intermediate" values of partial success or failure.

signal threshold. But it is a also a curse: since productive investment responds sluggishly, the principal finds it optimal to raise the signal threshold when the project's cost increases, which lowers productive investment and increases the inefficiency of the allocation.

We then extend our analysis to a dynamic setting in which the agent allocates the budget incrementally, one unit at a time, and observes the realization of the acquired signals before deciding whether to invest the next budget unit or use it to acquire a signal. This allows the agent to engage in experimentation and learning about the state through signal acquisition. However, we show that experimentation has no value in this setting. The main reason behind this is that, abstracting from the persuasion value of a signal, acquiring a signal for learning and experimentation is dominated by investing the same budget unit productively. Indeed, a unit of investment increases the expected project return, while an unbiased signal has zero expected effect on the project's return and, in the best case when it is a success, increases the project return by the same amount as one unit of investment.

Of course, the agent needs to acquire signals to persuade the principal, but it is optimal for the agent to frontload all the investment. That is, her optimal strategy is to invest at first and then to switch to information acquisition. It then follows that the equilibrium allocation in the dynamic setting is the same as in the static one where the agent makes a one-time allocation decision: in both settings the agent allocates the same shares of budget to productive investment and to information acquisition. Otherwise, if the allocations in the two setting were different, it would be optimal for the agent to use the one that achieves a better payoff in both settings.

Acquiring information for learning and experimentation has no value in our context because the alternative, productive investment, has a positive effect on project return. This distinguishes our model from the existing models of experimentation where the alternative is obtaining a risk-free return from another source. We believe that this feature is not unique to our set-up, and our analysis has broader applicability in other economically interesting contexts.

Our characterization of the equilibrium allocation and its inefficiency in static and dynamic contexts pertains to settings in which the principal-agent relationship does not rely on any contractual commitments. It is natural therefore to inquire whether this inefficiency can be alleviated when the parties can make some kind of a commitment ex ante. To explore this, we extend our analysis to consider three commitment scenarios. The first one is the agent's commitment to a full signal disclosure. The second is the principal's commitment to a decision rule, and the third is the combination of these two commitments.

Intuitively, by committing to full signal disclosure the agent ties his own hands. Hence, by obtaining and disclosing a small number of signals the agent can now convey to the principal that the rest of his budget has been invested. There is, however, one caveat to this rule that implies that the principal exhibits some skepticism and does not simply deduct the number of disclosed signals from the agent's budget to compute the productive investment. If the principal did this, then the agent would shirk and save the cost associated with productive investment which is small but positive in our model.[3] So, to be certain that the agent had productively invested the budget remaining after information acquisition, the principal requires at least one successful signal.

Consequently, in the setting with commitment to signal disclosure, the agent acquires such number of signals that the investment of the remaining budget together with one successful signal are sufficient to push the principal's expectation of the return above the cost. Therefore, as the project cost increases, the agent acquires less signals and increases productive investment. In fact, under a high project cost the budget allocation is close to the efficient one. This result stands in a sharp contrast to the outcome with no commitment where productive investment decreases in the project cost.

The principal's commitment to a decision rule results in an intermediate level of productive investment: below its level under the agent's commitment, but above the level in the setting without commitment. The principal optimally commits to a minimal signal threshold. Depending on the parameters, she requires either one or two successful signals for the project approval. This low threshold shifts the agent's incentives towards more productive investment. Still, in order to improve his chances of obtaining a successful signal, the agent allocates substantial resources to information acquisition, so the inefficiency persists. Since the principal's approval threshold does not change with the project cost, the agent's investment in productive activities does not change with that cost either.

Finally, we show that if the agent can commit to full signal disclosure, the principal does not get any additional benefit from her ability to commit to an approval threshold. In fact, the principal's additional commitment to an approval threshold would be counterproductive as it would undermine the effect of the agent's commitment to signal disclosure. Under the latter, the agent makes a large productive investment because the alternative – acquiring a lot of information which becomes observable– reveals low productive investment. This causes the principal to increase the threshold number of successful signals required for project approval. This effect is crucial for stimulating investment. Yet, it would disappear if the principal additionally committed to the approval threshold.

Our model has several applications. First, consider project approval process in organizations. The top management's decisions which projects to approve and which to abandon are a key factor in the success of an organization (Robert Gibbons, Niko Matouschek and John Roberts, 2012). Yet, there are typically no formal mechanisms for such approval. So a manager of a group developing a new technology typically has to produce substantial evidence to convince the top executives to support the project. Producing such evidence requires significant time and effort, which alternatively could be spent on improving the technology. While improving the technology also makes it easier to produce favorable evidence about it, our results

[3]The assumption that the agent bears a small fixed cost of productive investment is plausible, since productive investment typically requires not only a budget allocation but also additional inputs such as management, monitoring, etc.

show that the manager would overinvest in persuasion in the absence of clear commitments and approval conditions.[4]

As a second example, consider promotion process in organizations. Prior to the promotion decision, an employee allocates her time between investing in her fundamental skills that raise her productivity and self-promotion that signals her productivity. The latter may involve participating in conferences, making presentations, preparing publications, etc. Investing resources in the fundamental skills may be a more efficient to raise the employee's productivity. However, the managers cannot accurately measure such investment, and so promotion decision is typically affected by the employee's success in self-promotion. This stimulates the employee to overinvest in self-promotion activities, even though better fundamental skills make self-promotion more successful, [5]

Third, consider the development and approval process for new drugs and pharmaceutical products, such as the new COVID-19 vaccines. Besides financial investment, time is am essential resource in this process, since the vaccine has to be brought to market in a relatively short time period. A pharmaceutical company has to allocate available limited time to development and trials. Our results suggest that this allocation will be skewed towards trials. However, a commitment by the decision-maker to the approval threshold and the commitment by the firm to full disclosure of trial outcomes would improve the efficiency. Thus, our paper supports setting clear and unambiguous approval rules by the FDA as well as transparency of trials requiring pharma companies to register all their trials in advance.

Related Literature: There is a large literature on disclosure games starting with Sanford J Grossman (1981) and Paul R Milgrom (1981) who pioneered the study of strategic disclosure of verifiable information an agent holds verifiable information in an agency relationship. These authors as well as other earlier papers in this literature, including Ronald A Dye (1985); Michael J Fishman and Kathleen M Hagerty (1990); Woon-Oh Jung and Young K Kwon

[4]The history of Xerox corporation in the 1970s demonstrate that persuasion activities may crowd out valuable productive investment resulting in the rejection of potential technological breakthroughs. At the end of 1960s the first prototype of a personal computer was developed at Xerox's Palo Alto Research Center (PARC). However, the management of Xerox remained skeptical about the new technology, and so PARC employees spent a lot of resources on persuasion activities, which included numerous product presentations, training sessions for Xerox' top executives, preparation of long reports and memoranda and even installing the computers in the White House and the Senate, where they performed well (John Laprise, 2009; Natalya Vinokurova and Rahul Kapoor, 2020). All these efforts required diverting substantial resources from R&D, and the technology remained very expensive. In the end, Xerox executives abandoned this project, even through its core technology was quite revolutionary. Soon after that, top scientists and engineers left Xerox for other corporations, including Microsoft and Apple, where they helped to successfully commercialize the ideas developed at PARC. In particular, PARC scientists played central roles in developing Lisa and Macintosh personal computers at Apple. (Michael Hiltzik, 1999). Steve Jobs remarked later that "Xerox could have owned the entire computer industry today" (Omar A Nayeem, 2017).

[5]There is rich evidence that promotion decisions in organizations heavily rely on subjective performance evaluations that are affected by the employees' influence activities. The evidence also suggests that the employees engage in significant influence activities when a promotion is on the agenda (Jasmijn C Bol, 2008; Dennis Campbell, 2008; Chad A Higgins, Timothy A Judge and Gerald R Ferris, 2003; Rebecca A Thacker and Sandy J Wayne, 1995).

(1988); Masahiro Okuno-Fujiwara, Andrew Postlewaite and Kotaro Suzumura (1990); Robert E Verrecchia (1983), focused on the "unraveling" phenomenon resulting in all types of a sender fully revealing themselves in equilibrium. Our paper is more closely related to the subsequent literature that studies situations without full unraveling e.g., Viral V Acharya, Peter DeMarzo and Ilan Kremer (2011); Jacob Glazer and Ariel Rubinstein (2004, 2006); Ilan Guttman, Ilan Kremer and Andrzej Skrzypacz (2014); Barton L Lipman and Duane J Seppi (1995). However, this literature does not consider the tradeoff in the allocation of resources between productive and persuasion-related activities.

The interaction between disclosure and investment is studied in Anne Beyer and Ilan Guttman (2012). In their paper a manager privately observes the value of the firm and undertakes productive investment. The manager can then disclose the investment level to the market. They show that the manager undertakes a suboptimal investment that she publicly discloses in an attempt to distort the market's beliefs about the firm's value. In contrast to Beyer and Guttman (2012), in our paper the agent faces a different tradeoff of allocating resources between investment and information acquisition used for persuasion.

Elchanan Ben-Porath, Eddie Dekel and Barton L Lipman (2017) and Peter M DeMarzo, Ilan Kremer and Andrzej Skrzypacz (2019) study a disclosure setting where an agent chooses a distribution over outcomes and then decides whether to disclose the outcome to an outside observer. Ben-Porath, Dekel and Lipman (2017) show how the agent's control of information leads to inefficient risk-taking. In an attempt to impress an outside observer, the agent chooses a risky project even if the safer alternative has a higher expected value. They show that the agent only discloses sufficiently good outcomes, and otherwise pretends to be uninformed. DeMarzo, Kremer and Skrzypacz (2019) study a setting in which a seller chooses a test for a product of unknown quality, and then decides whether to disclose the test result to a buyer. Similar to Ben-Porath, Dekel and Lipman (2017) they show that the seller has an incentive to run an inefficient test.

While in Ben-Porath, Dekel and Lipman (2017) and DeMarzo, Kremer and Skrzypacz (2019) the agent chooses a distribution over observable outcomes, in our setup the agent effectively chooses two interdependent distributions. His investment decision influences the unobserved distribution of the project returns, and both the investment and the information acquisition decisions determine the distribution of signals.

A related literature (Glazer and Rubinstein, 2004, 2006; Sergiu Hart, Ilan Kremer and Motty Perry, 2017) studies informational efficiency of disclosure strategies when the agent is endowed with verifiable information and decides which information to disclose to affect the principal's decision. This literature characterizes conditions under which the outcomes of an optimal mechanism are equivalent to equilibria of disclosure games. In particular, Glazer and Rubinstein (2006) demonstrate this in a setup where the principal's action is binary. Itai Sher (2011) establishes a similar result for general action sets of the principal, and shows that it holds as long as the principal's payoff is concave. In our setup where the principal's action set is binary, and the agent has state-independent preferences and possesses verifiable information, the outcome under commitment is strictly preferred by the principal to the outcome of the disclosure game. The difference between our paper and this literature is that in our model the agent's type/project return is endogenous and his budget allocation affects both his type as well as the information that he can convey to the receiver about his type.

The rest of the paper is organized as follows. Section I presents the model. Sections II-IV contain the analysis and our main results. Section V concludes, while all proofs are relegated to the Appendix.

I Setup

A principal (she) owns a project of a known cost $c \in [0, 1]$ and an unknown return. Before taking a decision whether to approve the implementation of the project, the principal hires an agent (he) to develop it.[6] Project development includes productive investment that improves the project's return and acquisition of verifiable information about the project's return.

The agent is endowed with a fixed budget of size $n \geq 2$ that he can allocate between investment and information acquisition as specified below. The principal does not observe how the agent allocates the budget. The fixed budget may represent limited time available to develop the project. Alternatively, the principal may have a fixed amount of monetary, human or other resources that she can dedicate to the project and endow the agent with. This assumption is made to simplify the exposition and to focus on the tradeoff in the resource allocation. In the Appendix we show that our main findings hold when the budget is chosen endogenously.

If the principal decides to implement the project, then her payoff is $\theta - c$ where θ is the realized project return, while the agent's payoff is normalized to 1. If the project is rejected, both players receive zero payoff. Thus, the principal wants to approve the project only if it is profitable, while the agent always wants the project to be implemented.

Budget allocation: We assume that the budget allocation process is lumpy. Each unit of investment and each signal require one budget unit. The agent can invest any amount $k \in \{1, .., n\}$ productively and spend the remainder on acquiring $r \leq n - k$ signals about the project return. Therefore, the agent's budget allocation strategy is a pair $(k, r) \in \{0, ..., n\}^2$ such that $k + r \leq n$.

The agent incurs a small fixed cost $b > 0$ when he chooses a positive level of investment.[7]

[6] As an example, the principal can be thought of as the top management of a corporation who hires a team of engineers to improve and test a production technology. The principal may not possess the necessary knowledge to execute the project, as in (Ricardo Alonso, Wouter Dessein and Niko Matouschek, 2008; Heikki Rantakari, 2008), or may lack time to work on it, as in (Oriana Bandiera, Luigi Guiso, Andrea Prat and Raffaella Sadun, 2011).

[7] This assumption reflects that investment is arguably a more complex activity than information acquisition,

To ensure that this fixed costs does not preclude positive investment we make the following assumption:

Assumption 1. *The fixed cost of investment b satisfies $b < \frac{n-2}{n(n+1)}$ for any $n \geq 3$.*

When the agent chooses zero investment, the project return θ is determined by a draw from the uniform prior $U[0, 1]$. Each unit of investment can be thought of as an experiment that produces an alternative technology and results in an additional draw from $U[0, 1]$. The highest draw is the best available technology, and so the realized return θ is the maximum of these $k + 1$ draws.

Thus, with investment k, the project return is distributed according to the cdf $F_k(\theta) = \theta^{k+1}$.[8] The players do not observe θ until the payoffs are realized.

Signals are binary. The realization of a signal is denoted by s and could either be a "success" ($s = 1$) or a "failure" ($s = 0$), with $Pr(s = 1|\theta) = \theta$. The obtained r signals constitute hard evidence set $S_r := \{s_1, .., s_r\}$.

Disclosure: The signals are verifiable information, so the agent can only hide signals, but cannot forge them. To formalize the disclosure process, let $j(S_r)$ be the number of successes in the evidence set S_r: $j(S_r) = \sum_{i=1}^{r} s_i$. The agent's disclosure to the principal can be represented by a message that contains pair of numbers, $m_{r'}^{j'} = (r', j')$, where r' is the number of disclosed signals and j' is the number of successes among the disclosed signals. Given the evidence set S_r, the set of feasible messages is $M(S_r) = \{(r', j')|r' \leq r, r' - r + j(S_r) \leq j' \leq j(S_r)\}$. The lower bound on j' comes from the fact the agent cannot disclose more failures than their actual number, $r - j(S_r)$, in the set S_r.

Timing: First, the agent chooses how to allocate the budget between investment and information acquisition. Second, the return θ is drawn, the signals are realized and are privately observed by the agent. Then the agent decides which signals to disclose. The principal observes the disclosed signals, and decides whether to approve the implementation of the project, after which the payoffs are realized. The timeline is shown in Figure 1. In our baseline scenario we consider a static, once-and-for-all budget allocation. Subsequently, we study a dynamic budget allocation process.

Equilibrium: We use the standard notion of perfect Bayesian equilibrium and focus on equilibria in pure strategies. The agent's resource allocation and disclosure strategies must be sequentially rational given the principal's belief and her approval strategy. The principal's approval strategy must be sequentially rational given the agent's strategy and the principal's

as the former requires more management and monitoring than the latter. Technically, it allows us to rule out uninteresting equilibria.

[8]Thus, the distribution of the project return after a larger productive investment first-order stochastically dominates the project return distribution under a lower investment.

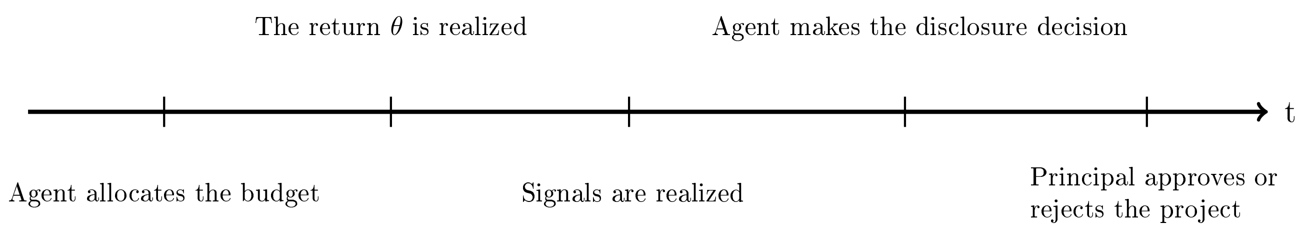


Figure 1: The Timing of the Game

beliefs, which are represented by a mapping from the set of disclosures into the set of probability distributions over $[0,1] \times \{0,...,n\}^3$, the product of the set of possible project returns, investment level k, the number of acquired signals r and the number of successes j. The beliefs are denoted by $\mu(m_{r'}^{j'})$ where $m_{r'}^{j'}$ stands for the agent's disclosure including r' signals and j' successes. The beliefs must be rational and consistent with the agent's resource allocation and disclosure strategy. That is, they must derived from the agent's strategy by Bayes rule on the equilibrium path. On an off equilibrium path, the beliefs must satisfy the restriction that $\mu(m_{r'}^{j'})$ puts a positive probability only on 4-tuples (θ, k, r, j) such that $k + r \leq n$, $r \geq r'$ and $j \geq j'$.

II Analysis and Main Results

A. Principal's beliefs and approval strategy

The principal approves the project if $E_P[\theta|\mu(m_{r'}^{j'})] \geq c$, and rejects it otherwise, where $E_P[\theta|\mu]$ is the principal's expectation of θ given her belief μ. Thus, it is useful to characterize the principal's beliefs about the project return θ. To do so, one can apply standard tools for our beta-Binomial model.[9] In particular, if the principal believes that the agent has made investment k, and obtained j successes in $r \geq j$ signals, her posterior beliefs about θ are characterized by the probability distribution with density

$$f(\theta|k,r,j) = \theta^{k+j}(1-\theta)^{r-j}\frac{(k+r+1)!}{(k+j)!(r-j)!}. \tag{1}$$

Therefore,

$$E_P[\theta|k,r,j] = \frac{k+j+1}{r+k+2}. \tag{2}$$

We now introduce some intuitive restrictions on the equilibrium strategies and beliefs. First, without loss of generality we focus on the agent's investment and information acquisition strategies (k,r) such that the agent spends the whole budget i.e., $k+r = n$. Acquiring $r < n-k$

[9]See Morris H DeGroot, Mark J Schervish, Xiangzhong Fang, Ligang Lu and Dongfeng Li (1986).

signals is weakly dominated by acquiring $n-k$ signals because (i) the principal does not observe the number of acquired signals; (ii) any disclosure that is feasible with r signals is also feasible with a larger number of signals. So the agent can never be better off by acquiring less than the maximal possible number of signals.

Next, we will restrict the principal's beliefs to have the following properties. Let k^* be the agent's equilibrium investment level.

Property 1. *(Uncontroverted k^* and Skepticism) If the agent discloses r' signals s.t. $r' \leq n - k^*$, then the principal's beliefs put probability 1 on the event that the agent has made investment k^*, has acquired $n - k^*$ signals, and failed to disclose $n - k^* - r'$ signals all of which are failures.*

Property 2. *Suppose that the agent discloses r' signals and j' successes s.t. $r' > n - k^*$. Then the principal's beliefs put probability 1 on the event that the agent has invested at most $n - r'$, and has disclosed all successes.*

Property 1 implies that the agent cannot convince the principal that he has invested more than the equilibrium level k^* by disclosing $r' < n - k^*$ signals. Indeed, disclosing such lower number of signals r' is still consistent with investment k^*. Therefore, upon observing $r' < n - k^*$ signals the principal maintains her equilibrium belief that the agent has invested k^*, and adopts a skeptical point of view that the "missing", undisclosed $n - k^* - r'$ signals are all failures.

Property 2 implies that upon observing $r' > n - k^*$ signals, the principal concludes that the agent has deviated from k^* to a lower investment level. Yet, she is still skeptical with regards to the number of successful signals and believes that all successes have been disclosed.

Together, Properties 1 and 2 imply that the principal believes that all successful signals are disclosed by the agent. This yields the following Lemma.

Lemma 1. *Let k^* be the agent's equilibrium investment level. Suppose that the agent discloses r' signals including j' successes, and that the principal's beliefs satisfy Properties 1 and 2,*

1. *If $r' \leq n - k^*$, then the principal's posterior expectation of θ is $\frac{1+j'+k^*}{n+2}$, and so she approves the project if and only if $\frac{1+j'+k^*}{n+2} \geq c$.*
2. *If $r' > n - k^*$, then the principal's posterior expectation of θ is bounded above by $\frac{n-r'+j'+1}{n+2}$, and so she approves the project only if $\frac{n-r'+j'+1}{n+2} \geq c$.*

Lemma 1 implies that the principal's equilibrium threshold j^* i.e., the minimal number of successes that she requires to approve the project, is

$$j^* = \lceil c(n+2) \rceil - (k^* + 1), \tag{3}$$

where k^* is the equilibrium level of investment. The decision rule (3) highlights that the principal lowers her evidence threshold when equilibrium investment level increases. It also reflects that a disclosed success is a perfect substitute for a unit of investment from the principal's perspective. Indeed, each affects the principal's posterior beliefs of θ in the same way.

B. Disclosure and budget allocation

Consider now the agent's optimal strategy. We start with her disclosure decision when the principal's beliefs satisfy Properties 1 and 2 and hence the threshold number of successful signals required for project approval is given by (3). Recall that given the equilibrium investment level k^*, the number of signals is $r^* = n - k^*$ since the agent would always exhaust the entire budget.

Lemma 2. *Suppose that the agent obtains r' signals of which j' are successes.*

(i) If $r' \leq r^*$, then it is optimal for the agent to disclose all signals.

(ii) If $r' > r^*$, then it is optimal for the agent to disclose $r^* = n - k^*$ signals and $\min\{j', r^*\}$ *successes.*

After any optimal disclosure, the principal believes that the agent has made investment k^* with probability 1.

To understand the Lemma, note the following. If the agent acquires less than r^* signals, he can never convince the principal that he has invested more than k^*. Therefore, it is optimal for the agent to disclose all successful signals. On the other hand, if the agent has deviated and acquired more signals (i.e. $r' > r^*$), then the principal's skepticism prevents the agent from disclosing more than r^* signals even if he obtained more than r^* successes.

Next, we consider the equilibrium budget allocation. To begin, we provide the first-best budget allocation benchmark which the principal would choose if she could allocate resources herself.

Lemma 3. *Suppose that the principal can choose the budget allocation. If $c \leq \frac{n+1}{n+2}$, she would spend the entire budget on investment and approve the project with probability 1. If $c > \frac{n+1}{n+2}$, the principal makes zero investment and never approves the project.*

To understand Lemma 3 note that each additional unit of investment shifts the distribution of the project return θ to the right. In contrast, in expectation a signal does not affect the distribution of the project return. Hence, the principal's first-best allocation is to invest the entire budget. Since this allocation maximizes the probability of the project approval, it would also be chosen by the agent if her strategy was observable.

This result comes with a qualifier that the project cost c cannot be too high. In particular, it cannot exceed $\frac{n+1}{n+2}$. This is because under the best possible scenarios when either all budget is productively invested or all obtained signals are successes, the posterior expectation of θ equals $\frac{n+1}{n+2}$. So the principal never approves the project if c exceeds this level. The next Lemma shows that the first-best allocation cannot arise in our setting where budget allocation is unobservable.

Lemma 4. *There is no equilibrium in which $k^* = n$.*

Suppose to the contrary that in equilibrium the agent invests all budget. Then the principal does not expect any signal disclosure, and the agent has a profitable deviation to $k = 0$ which saves her the fixed cost $b > 0$ associated with productive investment. Therefore, $k^* = n$ cannot be sustained in equilibrium.

Next, we focus on the agent's equilibrium budget allocation strategy and provide our central qualitative result. Our previous analysis implies that the agent's equilibrium investment k^* must be a solution to the following maximization problem:

$$\max_{k' \in \{0,..,n-j^*\}} Pr(j \geq j^*|k', n) - \mathbf{1}_{k'>0} b \tag{4}$$

where $j^* = \lceil c(n + 2)\rceil - (k^* + 1)$ is the signal threshold i.e., the minimal number of successful signals required by the principal for project approval when she expects the agent to invest k^* (see (3) and Lemma 1), and $Pr(j \geq j^*|k', n)$ is the probability of obtaining at least j^* signals given investment k'. The second term in (4) reflects the cost $b > 0$ that the agent incurs when she makes a positive investment.

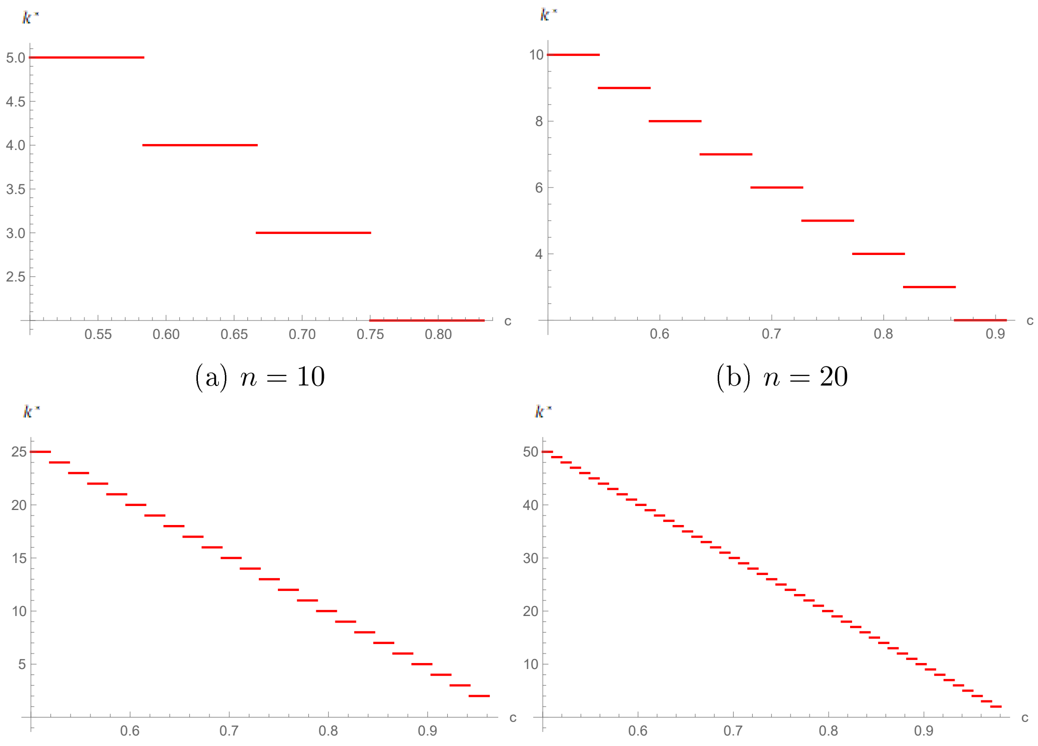
The analysis of the problem (4) underlies the next two results, Theorem 1 and Proposition 1. Theorem 1 presents our main qualitative result establishing that the level of information acquisition increases at the expense of productive investment as the project cost c increases. So the inefficiency of the budget allocation increases in the project cost. As we show below, this result is robust and holds both under a sequential budget allocation procedure, as well under more general preferences.

Theorem 1. *Suppose that $n \geq 3$ and $c \in [\frac{1}{2}, \frac{n}{n+2}]$. If the project cost c increases, the equilibrium level of investment decreases.*

Theorem 1 follows directly from Proposition 1 which characterizes the equilibrium allocation for different values of n and cost c.

Proposition 1. *The equilibrium budget allocation k^* and the principal's evidence threshold j^* are as follows:*

1. *If $c > \frac{n+1}{n+2}$, then $k^* = 0$ and the principal never approves the project.*
2. *If $c \in \left(\frac{n}{n+2}, \frac{n+1}{n+2}\right]$, then $k^* = 0$ and $j^* = n$.*[10]
3. *If $n > 3$ and $c \in \left(\frac{1}{2}, \frac{n}{n+2}\right]$ for even n and $c \in (\frac{n+3}{2(n+2)}, \frac{n}{n+2}]$ for odd n, then $k^* = (n + 2) - \lceil c(n + 2)\rceil$ and $j^* = 2\lceil c(n + 2)\rceil - (n + 3)$.*
4. *If n is odd, $n \geq 3$, and $c \in (\frac{n+1}{2(n+2)}, \frac{n+3}{2(n+2)}]$, then $k^* = \frac{n-1}{2}$ and $j^* = 1$.*



(a) $n = 10$ (b) $n = 20$

(c) $n = 50$ (d) $n = 100$

Figure 2: The equilibrium investment k^* for $n = 10, 20, 50, 100$ in the interval $c \in \left[\frac{n+4}{2(n+2)}, \frac{n}{n+2}\right]$

The equilibrium allocation is illustrated in Figures 2 and 3. Figure 2 shows that the equilibrium investment is around 50% of the budget when c is close to $\frac{1}{2}$, but decreases monotonically in the project cost and reaches a negligible level at high c. At the same time, Figure 3 highlights that the signal threshold j^* grows faster than investment decreases. Intuitively, as c grows, the principal must have a more favorable belief about the return θ to approve the project. She can achieve this only by increasing the signal threshold j^*, but this causes a decrease in investment.

The restrictions on costs in Proposition 1 stem from the fact that an equilibrium supported by weakly skeptical beliefs, which we focus on, fails to exist when the project cost is sufficiently small. Indeed, the condition for the agent not to deviate to a higher investment under weakly skeptical beliefs is $k^* \geq n - \lceil c(n+2) \rceil$, while the requirement $j^* \geq 1$ holds if $k^* \leq \lceil c(n+2) \rceil - 2$.

These two conditions are obviously incompatible when c is sufficiently small that $n + 2 > 2\lceil c(n + 2) \rceil$. However, for this cost range there exists an equilibrium in which the principal approves the project with probability 1. When the cost is intermediate ($\lceil c(n+2) \rceil < \frac{n}{2} + 1$ and $c > \frac{1}{n+2}$) the equilibrium supporting this outcome is quite unintuitive: it involves the agent using a weakly dominated strategy of not spending any budget (zero investment and no signal acquisition) and the principal forming beliefs that do not satisfy our skepticism restriction. On the other hand, when $c \leq \frac{1}{n+2}$, every equilibrium outcome is such that the principal approves the project with probability 1, because the principal's expectation of θ in the worst case of zero

[10]This case includes $n = 2$ and $c \in \left(\frac{1}{2}, \frac{3}{4}\right]$. So with $n = 2$ the equilibrium investment is always zero.

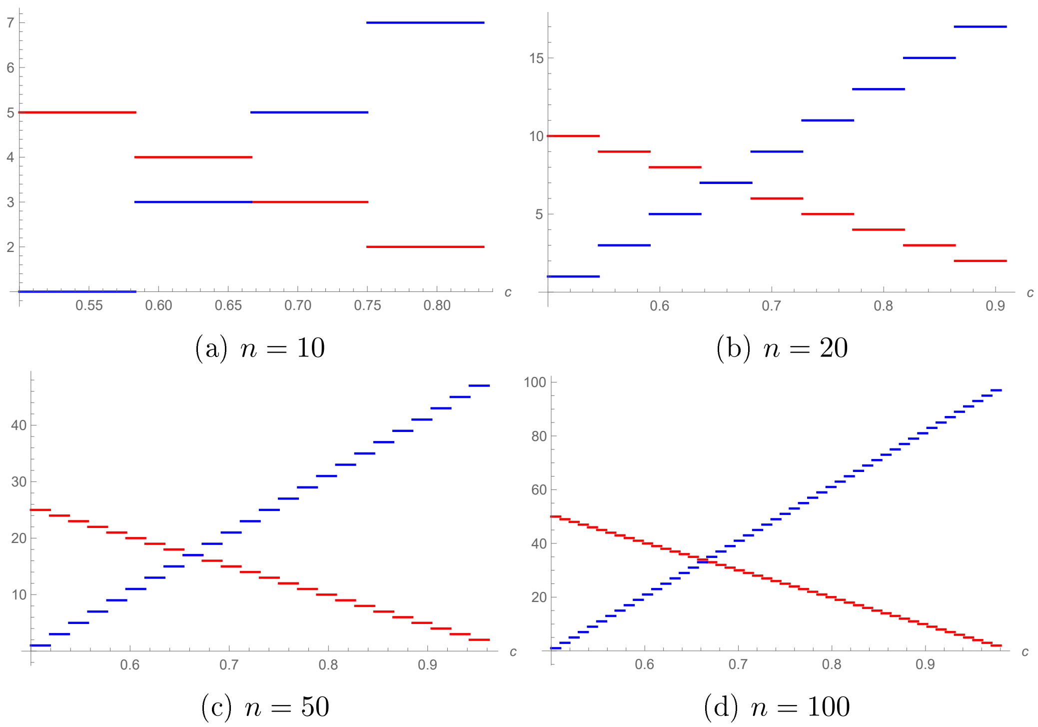


Figure 3: k^* decreasing in c, and j^* increasing in c, for $n = 10, 20, 50, 100$ and $c \in [\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$

investment and n failed signals is $\frac{1}{n+2}$.[11]

Finally, it is worth noting that the central result of this section -that the investment decreases in the project cost – does not depend on the fact that a successful signal and one investment unit have the same effect on the expected project return. We confirm this in the Appendix where we study a more general investment technology, specifically, such that investment k induces the probability distribution of the project return $\theta^{\rho k+1}$ where $\rho \in [\frac{1}{2}, 1]$.

III Sequential Budget Allocation

So far we have studied a setting in which the agent decides how to allocate the whole budget once and for all. However, some allocation processes have a sequential nature: the agent allocates the budget incrementally and observes some interim information before deciding how to allocate the next budget unit. In this section, we explore such scenario.

Specifically, suppose that in each time period $t \in \{1, ..., n\}$ the agent decides whether to invest one budget unit or use it to acquire an additional signal. If he acquires a signal at time period $t \in \{1, .., n-1\}$, then he observes the realization of this signal before the start of the next period $t+1$. We assume that the agent always perfectly recalls the history, including the allocation and signal realizations. We maintain the distributional assumptions of the baseline model, including the binary signal structure. Recall that these assumptions imply, in particular,

[11] As n grows large, $\frac{1}{n+2}$ converges to 0, and so for large budgets the project is approved automatically only when the cost c is very small.

that a budget unit invested productively shifts an observer's beliefs about the distribution of θ in the same way as a successful signal. By Bayes rule, her beliefs about the state θ at period t depend only on the numbers of invested units and realized successes, and not on their order.

Importantly, under sequential budget allocation the agent updates her beliefs about the value of θ in every period, and can employ a dynamic strategy rule prescribing whether to acquire signals or to invest depending on the past realizations of the signals. This allows her, in particular, to engage in some form of experimentation by acquiring signals and conditioning her subsequent choices on signal realizations. However, we show that the agent does not benefit from these dynamic possibilities. Her optimal strategy implements the same allocation as in the static case, as the next result demonstrates.

Theorem 2. *Suppose that $n \geq 2$ and $c \in \left(\frac{1}{2}, \frac{n}{n+2}\right]$. Then an optimal sequential budget allocation strategy for the agent is to invest k^* units in the first k^* periods and then to obtain $n - k^*$ signals in subsequent time periods.*

This optimal budget allocation is unique provided that the agent acquires a signal in every time period when she is indifferent between acquiring a signal and investing.

Theorem 2 says that the agent frontloads investment and then uses all the remaining budget to acquire signals. To understand why this is so, recall that investment has a positive effect on the project return; a successful signal has the same positive effect on the beliefs about the return as a unit of investment, while an unsuccessful signal negatively affects the beliefs about the project return. So there is no reason to acquire a signal for learning or experimentation, as it is better to invest instead. In fact, since the only value of a signal lies in its use as evidence to persuade the principal, we show that there exists a threshold such that the agent never acquires a signal if the probability of success is below this threshold. This implies that the agent needs to start with allocating the budget to investment for the first few periods.

On the other hand, once the agent started optimally acquiring signals, he never switches back to investment. Indeed, if that was ever to occur, the agent would be better off shifting such investment forward and shifting the information acquisition to a later stage. It follows that the optimal strategy consists of two stages: an investment stage that occurs over k' initial periods followed by information acquisition stage, without switching back and forth between the two activities. It then follows that the investment stage must be exactly k^*-long. Otherwise, i.e. if $k' \neq k^*$, k^* could not be optimal in the static case, for the agent in this case could get the same payoff as in the dynamic case by making an investment k^*, and vice versa.

A conclusion that emerges from the analysis of the sequential case is that the agent sees no point in experimenting in our setting, which ultimately determines why the sequential and static allocations coincide. As emphasized above, the experimentation is not valuable here because a successful signal changes the posterior beliefs about the project return in the same way as one unit of investment. This is a special feature of our environment. However, we believe that it is not unique to our set-up and also holds in other contexts and therefore our conclusion has a broader appeal and applicability.

IV Contractual Commitment

In this section we analyze the static setting with an additional element: either one or both players can commit to a particular course of action, or a decision rule. So in such set-ups the parties possess intermediate contractual opportunities: lesser than under the mechanism design paradigm which allows for full commitment and contingent transfers, bur richer than in the baseline no-commitment signaling and persuasion approach. The motivation behind this approach, as expressed by Nahum Melumad and Toshiyuki Shibano (1991), comes from the fact that it is common to find "intrafirm, regulatory, and political relationships where[in] an uninformed decision maker, attempting to elicit information from an informed party affected by his decisions, in unable to use transfers." These authors marry strategic communication setting of Vincent P Crawford and Joel Sobel (1982) with mechanism design without transfers, which is equivalent to the principal's commitment to a decision rule as a function of the agent's message.[12]

Studying contractual commitments without transfers in our setting is particularly natural and interesting: it provides an opportunity to consider and compare several forms of commitment, and to explore whether and to what extent different forms of commitment allow to curtail the inefficiency of the allocation. The first form of commitment that we study is the one by the agent to full disclosure. The second is the commitment by the principal to the decision rule. The third form is a cap on information acquisition set by the principal.

First, let us suppose that the agent can commit to signal disclosure obliging her to disclose all acquired signals and their outcomes. The outcome in this case is characterized in the following Proposition.

Proposition 2. *Suppose that the agent commits to reveal all signal realizations and that* $n \geq 2$*. Then, for* $c \in [\frac{1}{2}, \frac{n+1}{n+2}]$*, in equilibrium the agent chooses investment level* $k^* = \lceil c(n+2) \rceil - 2$ *and obtains* $n - k^*$ *signals, and the principal approves the project if the signals include at least one success i.e.,* $j^* = 1$.

The key aspect of the agent's commitment is that it allows him to credibly signal his investment choice to the principal. The disclosure of a certain number of signals indicates that the remainder of the budget has been invested productively.

There is, however, one caveat to the credibility of such message: the principal has to be sure that the agent did not shirk and invested nothing. For this reason, the principal requires the agent to deliver at least one successful signal. Otherwise, if no successes were required for the approval, the agent would deviate and choose zero investment to save the fixed cost b.

[12]Earlier contributions (Bengt Holmstrom, 1977, 1984) lay out the motivation and the framework for the study of commitment without transfers in organizations, which was later developed by the large literature on delegation.

But the agent does not want the principal to set a higher signal threshold, as it would make getting approval more difficult. So, the agent chooses to acquire a sufficiently small number of signals, which makes the principal believe that most of the budget has been invested productively and require just one successful signal for approval.

According to Proposition 2, the agent's investment increases in the project cost. This is so because approval under a higher cost requires either a larger investment or a higher number of successful signals, or both. So, in order to keep the principal's signal threshold at exactly 1 ($j^* = 1$), the agent increases investment and decreases information acquisition as the project cost goes up.

Next, let us consider the principal's commitment to a decision rule. By such commitment, the principal sets the threshold for project approval, j^c, ex ante. The decision rule must have a threshold nature, as otherwise the agent would simply withhold successful signals.

When the principal commits to the threshold j^c, the agent's best response it to invest $k(j^c) = \lfloor \frac{n-j^c+1}{2} \rfloor$ and to allocate the rest of the budget to information acquisition (see the proof of Proposition 3 for details). So the principal's optimal commitment threshold j^c solves the following program:

$$\max_{j' \in \{1,..,n-1\}} \sum_{j=j'}^{n-k(j')} Pr(j|k(j'), n)\Big(\frac{k(j') + j + 1}{n+2} - c\Big), \tag{5}$$

Solving this problem yields the following Proposition.

Proposition 3. *Let $n \geq 3$ and $c \in [\frac{1}{2}, \frac{n+1}{n+2}]$ and suppose that the principal can commit to approval threshold.*

The principal's equilibrium commitment threshold is $j^c = 1$ if n is even and $j^c = 2$ if n is odd. The agent invests $k = \lceil \frac{n-1}{2} \rceil$.

The key factor driving the result of Proposition 3 is that the agent's optimal investment level is decreasing in the principal's approval threshold. Hence, to induce a larger investment the principal commits to a low approval threshold, even though this may hurt the principal ex-post when the number of realized signal successes is low.

At the same time, Proposition 3 illustrates the limits of the principal's commitment power: the principal cannot induce the agent to spend more than half of his budget on investment. So, there remains a considerable gap between the outcome under the principal's commitment and the first-best allocation of all budget to investment.

The comparison of the agent's and the principal's payoffs under different commitment scenarios, and in the baseline no-commitment is provided in the following Proposition.

Proposition 4. *Suppose that $c \in [\frac{n+4}{2(n+2)}, \frac{n+1}{n+2}]$ and $n \geq 3$.*

(i) The principal prefers agent's commitment to her own commitment, and her own commitment to the equilibrium outcome without commitment.

(ii) The agent prefers both principal's commitment and his own commitment to the equilibrium outcome without commitment.

It is of course interesting to inquire whether a combination of commitments -considered so far separately -can be used to achieve a better outcome. To highlight this issue, we provide a result showing that the principal does not get any additional benefit from her own ability to commit to a decision rule when the agent can commit to full signal disclosure.

Corollary 1. *Consider $c \in [\frac{1}{2}, \frac{n+1}{n+2}]$ and suppose that the agent is able to commit to full signal disclosure. Then the principal's payoff and the investment level are higher when the principal does not commit to a decision rule than under the principal's commitment to a decision rule.*

The intuition behind this Corollary is as follows. When the principal commits to an approval threshold, only disclosed successful signals affect the outcome, and therefore the agent's investment is the same as under the principal's commitment only, which is less than the investment under the agent's commitment. So, the principal's additional commitment to a decision rule undermines strong investment incentives that the agent has under her own commitment to disclosure. Therefore, an additional commitment to an approval threshold does not benefit the principal when the agent can commit to signal disclosure.

In the last part of this section we extend the analysis of commitment in another direction by considering the situation where the principal can impose a restriction on the agent's budget allocation. This possibility appears to be quite intuitive. For example, top managers in an organization typically have the authority to restrict budget decisions of lower-level managers.

To understand the effect of such restrictions, we consider a setting in which the principal can set a cap on information acquisition. Formally, she has an ability to fix an upper limit ℓ on the budget that the agent can spend on acquiring signals.[13] Besides being practically intuitive, this extension is interesting for another reason. Recall that the first-best solution involves allocating all budget to productive investment. This suggests that the principal may want to set the budget limit ℓ on information acquisition as low as possible. However, imposing a very low ceiling on information acquisition or prohibiting it outright is counterproductive. Indeed, with a low ceiling the project would be approved with very few or even no successful signal realizations. But since the agent's investment is not observable, this would exacerbate the moral hazard problem and result in a very low or zero investment.

In contrast, requiring the agent to deliver successful signal realization(s) has a positive incentive effect on investment which implies that the optimal budget cap on informational acquisition should not be too low. This intuition is behind the following Proposition which shows that the optimal budget cap is binding and is below the equilibrium level of information acquisition in the baseline no-commitment case characterized in Proposition 1.

[13] A commitment to exact information acquisition level would result in the same outcome since in equilibrium characterized in Proposition 5 the budget cap on information acquisition is binding.

Proposition 5. *Suppose that the principal can set a cap on the budget used by the agent for information acquisition and that* $c \in \left[\frac{n+3-\sqrt{n+3}}{n+2}, \frac{n}{n+2}\right]$. *The optimal such cap is* $\ell^e = n+2-\lceil c(n+2)\rceil$. *In equilibrium the cap is binding: the agent obtains* ℓ^e *signals and spends the remainder of the budget,* $\lceil c(n+2)\rceil - 2$, *on productive investment. The project is approved if the agent discloses at least one success i.e.,* $j^* = 1$.

Intuitively, the optimal budget cap on information acquisition, $\ell^* = n+2-\lceil c(n+2)\rceil$, is such that the corresponding approval threshold j^* remains positive when the agent invests the remaining budget $\lceil c(n+2)\rceil - 2$. This investment level and the associated principal's payoff are higher than in the equilibrium of the baseline no commitment scenario (Proposition 1). So, the principal benefits from his ability to restrict information acquisition. However, this beneficial effect is limited and a higher investment cannot be supported in an equilibrium, since it would cause the principal's threshold on the number of successful signals required for approval to become zero, which would in turn cause investment to fall to zero. This "unraveling" argument implies that a lower cap on information acquisition would undermine agent's incentives and result in zero investment.

The outcome with the optimal cap on information acquisition is the same as the outcome with the agent's commitment to disclosure, albeit under more restrictive cost conditions. Essentially, both commitment policies allow to implement the maximal possible investment level $\lceil c(n+2)\rceil - 2$. which is around a half of the budget when c is close to $\frac{1}{2}$, yet increases as cost c increases but never attains the first-best. A higher investment level cannot be achieved with any contract or commitment, because it would be inconsistent with positive approval threshold j which is necessary to provide the incentives for the agent to invest. So, inefficiency is fundamental to this problem.

.

V Conclusions

In this paper we have studied the interaction between productive investment and acquisition of information for persuasion. We have demonstrated that persuasion motive leads to an inefficient allocation of resources favoring information acquisition. Further, we have shown that the equilibrium under sequential resource allocation is the same as under a simultaneous one-time resource allocation procedure. The logic of this result suggests that experimentation that allows to learn about the quality of the project may not be valuable in a broad set of circumstances when the alternative to experimentation involves activities that improve the project's return and productivity.

The inefficiency of the allocation can be curtailed, albeit only partially, by the agent's commitment to information disclosure and the principal's commitment to a decision rule as a function of information disclosure.

Our analysis assumes that the agent's preferences are state-independent: the agent wants the principal to implement the project irrespective of the implementation cost. In Appendix B we consider a more general preference specification according to which the agent cares both about the project adoption and the net profit from the project. There are two takeaways from this analysis in the setting with static budget allocation. First, the equilibrium investment increases in preference alignment, and so a closer alignment of the players' preferences increases the efficiency of resource allocation. Second, the main message of Theorem 1 continues to hold if the preferences are not too aligned. In particular, even if the agent cares relatively strongly about the net profit, he has an incentive to acquire more signals as the project cost increases, especially when this cost is high. So the agent shifts resources towards persuasion and information acquisition when the project cost increases.

Another natural extension is to introduce a monetary transfer scheme in the principal-agent relationship. In particular, such transfers could be contingent on signal disclosure. Other avenues for future research involve studying the tradeoff between productive and informational activities in a framework with unverifiable information such as in Crawford and Sobel (1982) and in a setting where the agent can choose an information structure as in Kamenica and Gentzkow (2011).

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VI Appendix A

Proof of Lemma 1 If $r' \leq n - k^*$, then the principal's posterior beliefs about θ are characterized by the probability density function:

$$
\begin{aligned}
f(\theta|k^*, n-k^*, j') &= \frac{Pr(j'|n-k^*,\theta)(k^*+1)\theta^{k^*}}{\int_0^1 Pr(j'|n-k^*,\theta)(k^*+1)\theta^{k^*}d\theta} \\
&= \frac{\binom{n-k^*}{j'}\theta^{j'}(1-\theta)^{n-k^*-j'}(k^*+1)\theta^{k^*}}{\int_0^1 \binom{n-k^*}{j'}\theta^{j'}(1-\theta)^{n-k^*-j'}(k^*+1)\theta^{k^*}d\theta} = \theta^{k^*+j'}(1-\theta)^{n-k^*-j'}\frac{(n+1)!}{(k^*+j')!(n-k^*-j')!}.
\end{aligned} \tag{6}
$$

Correspondingly, the principal's expectation of θ is given by:

$$
\begin{aligned}
E_P(\theta|k^*, n-k^*, j') &= \int_0^1 \theta f(\theta|k^*, n-k^*, j')d\theta \\
&= \int_0^1 \theta^{k^*+j'+1}(1-\theta)^{n-k^*-j'}d\theta \frac{(n+1)!}{(k^*+j')!(n-k^*-j')!} = \frac{k^*+j'+1}{n+2}.
\end{aligned} \tag{7}
$$

If $r' > n - k^*$, then the principal's posterior beliefs about θ are characterized by the probability density function:

$$
\begin{aligned}
f(\theta|m_{r'}^{j'}) &= \sum_{i=0}^{n-r'} \mu_w^i(r',j') \frac{Pr(j'|n-i,\theta)(i+1)\theta^i}{\int_0^1 Pr(j'|n-i,\theta)(i+1)\theta^i d\theta} \\
&= \sum_{i=0}^{n-r'} \mu_w^i(r',j') \frac{\binom{n-i}{j'}\theta^{j'}(1-\theta)^{n-i-j'}(i+1)\theta^i}{\int_0^1 \binom{n-i}{j'}\theta^{j'}(1-\theta)^{n-i-j'}(i+1)\theta^i d\theta} \\
&= \sum_{i=0}^{n-r'} \mu_w^i(r',j')\theta^{i+j'}(1-\theta)^{n-i-j'}\frac{(n+1)!}{(i+j')!(n-i-j')!}.
\end{aligned} \tag{8}
$$

Correspondingly, the principal's expectation of θ satisfies:

$$
\begin{aligned}
E_P(\theta|m_{r'}^{j'}) &= \sum_{i=0}^{n-r'} \mu_w^i(r',j') \int_0^1 \theta^{i+j'+1}(1-\theta)^{n-i-j'}d\theta \frac{(n+1)!}{(i+j')!(n-i-j')!} \\
&= \sum_{i=0}^{n-r'} \mu_w^i(r',j') \frac{i+j'+1}{n+2} \leq \frac{n-r'+j'+1}{n+2}.
\end{aligned} \tag{9}
$$

Q.E.D.

Proof of Lemma 2: First, note that the agent's optimal disclosure strategy maximizes the principal's posterior expectation of θ given by (7) if $r' \le n - k^*$ or (9) if $r' > n - k^*$.

Let us start with case (i) in which the agent acquires (weakly) less than the equilibrium number of signals $r^* = n - k^*$. Then, no matter what he discloses, according to Property 1 the principal believes that with probability 1 the agent has invested k^* units.

So, if the agent discloses all signals, which includes j' successes, then according to (7) the principal's posterior expectation of θ is equal to $E_P(\theta|k^*, n-k^*, j') = \frac{k^*+j'+1}{n+2}$,

On the other hand, if the agent makes a disclosure $m_{r''}^{j''}$ that includes j'' successes and r'' signals where $j'' \le j'$ and $r'' \le r'$ then, again by (7), the principal's posterior expectation of θ is equal to $E_P(\theta|k^*, n-k^*, j'') = \frac{k^*+j''+1}{n+2}$. Since the latter is less than $E_P(\theta|k^*, n-k^*, j') = \frac{k^*+j'+1}{n+2}$ as $j'' \le j'$, such deviation is not profitable.

Now consider case (ii) in which $r' > r^* = n - k^*$. If the agent chooses to disclose $r^* = n - k^*$ signals and $\min\{j, n-k^*\}$ successes, then according to Property 1 the principal believes that with probability 1 the agent has invested k^* units. So, by (7) the principal's posterior expectation of θ is equal to $E_P(\theta|k^*, n-k^*, j') = \frac{k^*+\min\{j',n-k^*\}+1}{n+2}$. Thus, to complete the proof we need to show that the principal's posterior expectation under any alternative disclosure strategy cannot exceed $\frac{k^*+\min\{j',n-k^*\}+1}{n+2}$.

First, if $j' \ge n - k^*$, then by (7) disclosing $r^* = n - k^*$ signals and $n - k^*$ successes induces the principal's posterior expectation equal to $\frac{n+1}{n+2}$. On the other hand, if the agent follows any alternative disclosure strategy $m_{r''}^{j''}$ s.t. $r'' > n - k^*$, then according to (9) the principal's posterior expectation of θ does not exceed $\frac{n-r''+j''+1}{n+2}$. The latter expression does not exceed $\frac{n+1}{n+2}$ since $j'' \le r''$. So deviating from disclosure $m_{n-k^*}^{n-k^*}$ is not profitable.

Next, suppose that $r' > n - k^*$ and $j' < n - k^*$. Then by making a disclosure $m_{r''}^{j''}$ where $r'' \le n-k^*$ and $j'' \le j'$, the agent induces the principal's posterior expectation equal to $\frac{k^*+j''+1}{n+2}$ by (7). The latter does not exceed $\frac{k^*+j'+1}{n+2}$, the principal's posterior expectation of θ when the agent discloses $n-k^*$ signals and j' successes. Therefore, such disclosure $m_{r''}^{j''}$ where $r'' \le n-k^*$ and $j'' \le j'$, is suboptimal.

Finally, suppose that the agent's disclosure $m_{r''}^{j''}$ is such that $r'' > n - k^*$ and $j'' \le j'$. Then by (9) the principal's posterior expectation does not exceed $\frac{n-r''+j''+1}{n+2}$ which, in turn, is less than $\frac{k^*+j'+1}{n+2}$. The latter is the principal's posterior after disclosure $m_{n-k^*}^{j'}$. So we conclude that the disclosure $m_{r''}^{j''}$ is suboptimal.

Finally, since the agent's optimal disclosure is always such that $r'' \le k^*$, it follows by Property 1 that the principal believes that the agent has invested k^* units with probability 1. *Q.E.D.*

Proof of Lemma 3: First, for $c > \frac{n+1}{n+2}$ there is no budget allocation resulting in $E(\theta|\cdot) > \frac{n+1}{n+2}$. So, the principal optimally chooses $k^* = 0$ and the equilibrium approval probability is zero. The principal is indifferent between acquiring any number of signals (conducting any number of trials) as no signal realization ever leads to the project approval: in the best possible scenario – when all trials are successes – the expected value of θ is $\frac{n+1}{n+2}$.

Consider now $c \leq \frac{n+1}{n+2}$. Suppose that the principal invests n units. Then, the expected payoff is given by $\frac{n+1}{n+2} - c \geq 0$. Now, suppose that the principal chooses $k < n$ invested units instead. Then, her expected payoff is

$$\sum_{j=0}^{n-k} Pr(j|k,n) \max\{0, [E(\theta|k,j,n) - c]\} = \sum_{j=0}^{n-k} Pr(j|k,n) \max\left\{0, \left[\frac{j+k+1}{n+2} - c\right]\right\}$$
$$< \sum_{j=0}^{n-k} Pr(j|k,n) \max\left\{0, \left[\frac{n+1}{n+2} - c\right]\right\} \leq \frac{n+1}{n+2} - c$$

Thus, the principal maximizes her payoff by choosing investment $k = n$. *Q.E.D.*

Proof of Lemma 4: Suppose that $c \leq \frac{n+1}{n+2}$ and the agent invests n in equilibrium. Then there is no budget left for signal acquisition i.e., $r = 0$, and there is no disclosure in equilibrium. Given that the principal believes that all budget has been invested, she approves the project. But then the agent has a profitable deviation to $k = 0$ which saves her the fixed cost of investment b, a contradiction.

Now suppose that $c > \frac{n+1}{n+2}$. In this case, $\mathbb{E}_P(\theta|k,n) \leq \frac{n+1}{n+2}$ under any budget allocation $(k, n-k)$, so the principal never approves the project. Hence, the agent's optimal strategy must involve choosing $k = 0$ to save the cost b. Thus, there is no equilibrium with $k^* = n$. *Q.E.D.*

Proof of Proposition 1:

At first, let us establish the following Lemma.

Lemma 5. *Let $Pr(j \geq j'|k,n)$ be the probability of obtaining at least j' successful signals under the allocation (k,n), where $j \leq n-k$. We have:*

$$Pr(j \geq j'|k,n) = 1 - \frac{(k+j')!(n-k)!}{(n+1)!(j'-1)!}. \tag{10}$$

Proof of Lemma 5:

First, let us compute $Pr(j|k,n)$, the probability of obtaining j successful signals under the allocation (k,n), where $j \leq n-k$. We have:

$$Pr(j|k,n) = \int_0^1 Pr(j|k,n,\theta)Pr(\theta|k)d\theta =$$
$$= \int_0^1 \binom{n-k}{j} \theta^j (1-\theta)^{n-k-j}(k+1)\theta^k d\theta = \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}.$$

Summing up yields:

$$Pr(j \geq j'|k,n) = 1 - \sum_{j=0}^{j'-1} Pr(j|k,n) = 1 - \sum_{j=0}^{j'-1} \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}$$
$$= 1 - \frac{(k+1)(n-k)!}{(n+1)!} \sum_{j=0}^{j'-1} \frac{(j+k)!}{j!} = 1 - \frac{(k+1)(n-k)!}{(n+1)!} \frac{(k+j')!}{(k+1)(j'-1)!} =$$
$$1 - \frac{(k+j')!(n-k)!}{(n+1)!(j'-1)!}, \tag{11}$$

where the second-to-last equality relies on the identity for the sum of partial factorials saying that $\sum_{j=0}^{j'-1} \frac{(j+k)!}{j!} = \frac{(k+j')!}{(k+1)(j'-1)!}$. This completes the proof of the Lemma.

Next, the equilibrium level of investment k^* solves the program (4). Let us at first ignore the fixed cost b and solve the following problem:

$$\max_{k \in \{0,..,n-j^*\}} Pr(j \geq j^*|k,n). \tag{12}$$

The solution k^* to this problem is such that the agent would never deviate from k^* to any other positive investment level. Then we show that provided that Assumption 1 holds, a deviation to zero investment is also unprofitable.

By equation (10) in Lemma 5, the objective in (12) is equal to:

$$1 - \frac{(k+j^*)!(n-k)!}{(j^*-1)!(n+1)!}, \tag{13}$$

Thus,

$$k^* \in \arg \min_{k \in \{0,...,n-j^*\}} (k+j^*)!(n-k)! \tag{14}$$

To solve the problem (14), let us first allow $k \in [0, n-j^*]$. Then the objective of (14) can be rewritten as

$$D(k,j^*,n) \equiv \Gamma(k+j^*+1)\Gamma(n-k+1) \equiv \int_0^\infty x^{k+j^*} e^{-x} dx \times \int_0^\infty x^{n-k} e^{-x} dx$$

Note that

$$\frac{dD(k,j^*,n)}{dk} = (\log(k+j^*) - \log(n-k))\,\Gamma(k+j^*+1)\Gamma(n-k+1) \tag{15}$$

It follows from (15) that $\frac{dD(k,j^*,n)}{dk} < 0$ ($\frac{dD(k,j^*,n)}{dk} > 0$) if $k < \frac{n-j^*}{2}$ ($k > \frac{n-j^*}{2}$) and $\frac{dD(k,j^*,n)}{dk} = 0$ if $k = \frac{n-j^*}{2}$. Hence, $D(k,j^*,n)$ attains a unique minimum at $k = \frac{n-j^*}{2}$. Also note that, for fixed (j^*,n), $D(k,j^*,n)$ is symmetric around $k = \frac{n-j^*}{2}$. Therefore, k^* minimizing (14) over

$\{0, ..., n - j^*\}$ satisfies:

$$\frac{n - j^* - 1}{2} \leq k^* \leq \frac{n - j^* + 1}{2}. \tag{16}$$

Since $j^* := \lceil c(n+2) \rceil - (k^* + 1)$ by Lemma 1, we can substitute this into (16) to obtain:

$$n - \lceil c(n+2) \rceil \leq k^* \leq n + 2 - \lceil c(n+2) \rceil. \tag{17}$$

Note that we must have $j^* \geq 1$ since an equilibrium with $k^* > 0$ and $j^* = 0$ cannot be supported because in this case the agent would deviate to save the fixed cost $b > 0$ of investment. Using (17), the condition $j^* \geq 1$ can be rewritten as:

$$k^* \leq \lceil c(n+2) \rceil - 2. \tag{18}$$

In the rest of the proof, we characterize equilibria for different cost ranges and $n \geq 3$.
Case 0. $c > \frac{n+1}{n+2}$**.** In this case, the principal never approved the project since $\frac{1+j+k}{n+2} < c$ for any j and k. Also, $k^* = 0$ by (17).
Case 1. either n is even and $c \in (\frac{1}{2}, \frac{n}{n+2}]$, or n is odd and $c \in (\frac{n+3}{2(n+2)}, \frac{n}{n+2}]$.

In this case, $n + 2 - \lceil c(n+2) \rceil \leq \lceil c(n+2) \rceil - 2$, so (18) holds if (17) holds. Therefore, the equilibrium k^* is determined only by (17). Since (17) has multiple solutions, we will focus on a Pareto efficient equilibrium with the highest investment where

$$k^* = (n+2) - \lceil c(n+2) \rceil.$$

Case 2. n is even and $c \in (\frac{n}{2(n+2)}, \frac{1}{2}]$. In this case, $n - \lceil c(n+2) \rceil = \frac{n}{2} - 1 = \lceil c(n+2) \rceil - 2 < n + 2 - \lceil c(n+2) \rceil$. Hence, there is a unique equilibrium with a positive investment $k^* = n - \lceil c(n+2) \rceil = \frac{n}{2} - 1$.
Case 3. n is odd and $c \in (\frac{n+1}{2(n+2)}, \frac{n+3}{2(n+2)}]$.

Note that $\frac{n+1}{2(n+2)} < \frac{1}{2}$. In this case, $n - \lceil c(n+2) \rceil = \frac{n-3}{2} < \frac{n-1}{2} = \lceil c(n+2) \rceil - 2 < \frac{n+1}{2} = n + 2 - \lceil c(n+2) \rceil$. So there are two values of k^* that satisfy both (17) and (18), of which we choose the larger one:

$$k^* = \lceil c(n+2) \rceil - 2 = \frac{n-1}{2}. \text{ Hence, } \quad j^* = 1.$$

Let us show that in Cases 1-3 the agent does not wish to deviate from k^* to $k = 0$ when Assumption 1 holds i.e., $b \leq \frac{n-2}{n(n+1)}$. In fact, we will show that the agent prefers investment $k = 1$, with $r = n - 1$ to $k = 0$ and $r = n$ under the same j, which then implies that she prefers k^* to $k = 0$.

By (10), the probability of obtaining at least j successes with $k = 0$ is $Pr(j' \geq j | k = 0, r = n - k) = 1 - \frac{j}{n+1}$, while the probability of obtaining at least j successes with $k = 1$, is

$Pr(j' \geq j|k=1, r=n-k-1) = 1 - \frac{(j+1)j}{(n+1)n}$. Then,

$$Pr(j' \geq j|k=1, r=n-1) - Pr(j' \geq j|k=0, r=n) = \frac{j(n-j-1)}{n(n+1)}. \tag{19}$$

Note that from the principal's best response (3), $c \leq \frac{n}{n+2}$ and $k^* \geq 1$ it follows that $j^* \leq n-2$. On the domain $j \in \{1, ..., n-2\}$, (60) reaches a minimum $\frac{n-2}{n(n+1)}$ both at $j=1$ and at $j=n-2$. Thus, if $b < \frac{n-2}{n(n+1)}$ and $n \geq 3$, the agent prefers investment $k=1$ to no investment, and so a priori she would not deviate from $k^* > 0$ to $k=0$.

Now consider $n=2$. In this case, the cost interval $(\frac{1}{2}, \frac{n+1}{n+2}]$ is $(\frac{1}{2}, \frac{3}{4}]$. The only candidate for an equilibrium with positive investment is $k=1$ and $j=1$. But in this case the value of (60) is zero, so the agent would choose $k=0, r=2$, instead. So, there is no equilibrium with a positive investment. However, we have an equilibrium $k^*=0, r^*=2$ and $j^*=2$. The latter holds by (3). Note that when $c=\frac{1}{2}$, we have an equilibrium $k^*=0$, $r^*=2$, $j^*=1$. *Q.E.D.*

Proof of Theorem 2.

The proof for the case $n=2$ is trivial, for in the sequential setting with $n=2$ the unique optimal action in the last period $t=2$ is to acquire a signal. Then investing in $t=1$ is optimal if and only if $k^*=1$ in the static setting.

So, for the rest of the proof we assume that $n \geq 3$. We start with several preliminary steps.

First, the principal's optimal decision rule after disclosure remains the same as in the simultaneous budget allocation case given by (3). So, if the principal believes that the agent has invested k^* and obtained $n-k^*$ signals of which d are successes, she approves the project if $d \geq j^* = \lceil c(n+2) \rceil - (k^*+1)$.

Further, the agent observes the history at any period $t \in \{1, ..., n\}$ which consists of the allocations made by the agent and the realizations of the acquired signals in every time period $s \in \{1, .., t-1\}$. Any two histories at period t that include the same total investment and the same number of successful signals result in the same agent's beliefs about θ, and so the agent's optimal strategy at any period t depends only on the "state" at time t, which is defined as a triple (k, d, r), where k is the total investment made before t, d is the number of successful signals and r is the number of unsuccessful signals acquired before t. Note that $k+d+r = t-1$ since one budget unit is allocated in every period.

If the state (k, d, r) associated with time period t is such that $k+r > n-j^*$, then $d < j^*$ and the agent does not have enough budget to acquire j^*-d signals necessary to persuade the principal. Then acquiring signals at any $t' \geq t$ is optimal, and any strategy is optimal if $k > 0$. The unique optimal strategy to acquire signals at all $t' \geq t$ if $k+r = n-j^*$ because in this case the agent can only persuade the principal if she spends the remaining budget j^*-d on signals and they all turn out to be successes.

Let us now focus on the case where the state is such that $k+r < n-j^*$. The proof proceeds through three claims. Claims 1 and 3 establish the Theorem. Claim 2 is an intermediate step to prove Claim 3.

Claim 1. *Suppose that the state (k,d,r) at time period t is such that $k \geq k^*$ and $k+r < n-j^*$. Then the agent's optimal continuation strategy is to acquire a signal at any $t' \in \{t, ...n\}$.*

Proof of Claim 1.

Let us define two continuation strategies starting from any time period $t \in \{1, ..n\}$. The continuation strategy σ_S^t prescribes that the agent acquires a signal in any period $t' \in \{t, ..., n\}$. The continuation strategy σ_I^t prescribes that the agent invests in period t and then acquires a signal in any period $t' \in \{t+1, ..., n\}$. Let us show that the agent's expected payoff from σ_S^t is greater than his payoff from σ_I^t.

Let $z = k - k^*$ and $s := k + d$. We refer to s as the number of positive outcomes. The agent's posterior belief given s positive outcomes and r signal failures is characterized by the following density:

$$f(\theta|s,r) = \frac{\binom{s+r}{s}\theta^e(1-\theta)^r}{\int_0^1 \binom{s+r}{s}\theta^e(1-\theta)^r d\theta} = \theta^e(1-\theta)^r\frac{(r+s+1)!}{r!s!}. \tag{20}$$

Since the agent's remaining budget at time t is $n-(s+r)$, by following strategy σ_S^t she obtains j successes with the following probability:

$$Pr(j|s,r) = \int_0^1 \binom{n-(s+r)}{j}\theta^j(1-\theta)^{n-(s+r)-j}f(\theta|s,r)d\theta = \frac{(j+s)!(s+r+1)!(n-j-s)!\binom{n-s-r}{j}}{(n+1)!r!s!}. \tag{21}$$

Next, consider continuation strategy σ_I^t. The posterior distribution of θ after the agent invests the $s+1$-th unit is characterized by the density: $f(\theta|s+1,r) = \theta^{s+1}(1-\theta)^r\frac{(r+s+2)!}{r!(s+1)!}$. So the probability of obtaining at least j successes under strategy σ_I^t is:

$$Pr(j|s+1,r) = \frac{(j+s+1)!(r+s+2)!(n-(j+s+1))!\binom{n-r-s-1}{j}}{(n+1)!r!(s+1)!}. \tag{22}$$

Strategy σ_S^t is more profitable than σ_I^t iff $\sum_{j=j^*-d}^{n-(s+r)} Pr(j|s,r) - \sum_{j=j^*-d}^{n-(s+r+1)} Pr(j|s+1,r) \geq 0$. To compute this difference, first, note that for $j \in \{j^*-d, ..., n-(s+r+1)\}$, we have:

$$\begin{aligned} &Pr(j|s,r) - Pr(j|s+1,r) = \\ &\frac{(j+s)!(r+s+1)!(n-(j+s+1))!\left((s+1)(n-j-s)\binom{n-r-s}{j} - (j+s+1)(r+s+2)\binom{n-r-s-1}{j}\right)}{(n+1)!r!(s+1)!}. \end{aligned} \tag{23}$$

From (23) it follows that:

$$\sum_{j=j^*-d}^{n-(s+r)} Pr(j|s,r) - \sum_{j=j^*-d}^{n-(s+r+1)} Pr(j|s+1,r) =$$

$$(j^* - n + r + 2s - d + 1)\frac{(r+s+1)!(j^*+s-d)!(n-(s+r+1))!(n-j^*-s+d)!}{(n+1)!r!(s+1)!(j^*-d-1)!(n-j^*-r-s+d)!}. \tag{24}$$

Since $r + k < n - j^*$, $n - j^* - r - s + d = n - j^* - r - k > 0$. Hence, the sign of the expression (24) is the same as the sign of $j^* - n + r + 2s - d + 1$. Further,

$$j^* - n + r + 2s - d + 1 = j^* + 2k^* - n + 2z + d + 1 + r \geq 2z + d + r \geq 0 \tag{25}$$

The first inequality in (25) holds because by (16) $j^* + 2k^* \geq n - 1$. So, strategy σ_S^t dominates the strategy σ_I^t for the agent, strictly if $z + d + r > 0$.

Let us now show that the optimal strategy requires the agents to acquire a signal in any time period t where the state is such that $k \geq k^*$. The argument is by induction. If the state (k, d, r) at $t = n$ is such that $k \geq k^*$, then (25) implies that at $t = n$ it is optimal to acquire a signal. Now consider $t = n - 1$. It is optimal to acquire a signal at $t = n - 1$ when the investment level k at $t = n - 1$ satisfies $k \geq k^*$ because σ_S^t dominates σ_I^t and at $t' = n$ it is optimal for the agent to acquire a signal.

Now consider time period $t < n - 1$ and suppose the investment level k at this time period is at least k^*, and the agent's strategy is to acquire signals at any $t' \in \{t + 1, n\}$. Since, as shown above, σ_S^t dominates σ_I^t at t, the optimal strategy it to acquire a signal at t. Hence, starting at any t with state of the world (k, d, r) s.t. $k \geq k^*$, the optimal continuation strategy it to acquire a signal at any $t' \in \{t, ..., n\}$. This establishes Claim 1. *Q.E.D.*

Claim 2. *Suppose that the state (k, d, r) at time period $t \in \{1, .., n - 2\}$ satisfies $d < j^*$, $k + d < k^*$, and $k + r < n - j^*$. Consider two continuation strategies. Under the first continuation strategy, $\sigma_1^{k,d,r}$, the agent invests at t and obtains a signal at any $t' > t$. The second continuation strategy, $\sigma_2^{k,d,r}$, prescribes to obtain a signal at t. If this signal is a success, the agent acquires a signal at any $t' > t$. If the signal acquired at t is a failure, the agent invests at $t + 1$ and then obtains a signal at any $t' \in \{t + 2, ..., n\}$.*

Let $\pi(\sigma_i^{k,d,r})$ denote the agent's expected payoff from the continuation strategy $\sigma_i^{(kd,r)}$, $i \in \{1, 2\}$ and $\Delta\sigma^{k,d,r} \equiv \pi(\sigma_1^{k,d,r}) - \pi(\sigma_2^{k,d,r})$.

For any k, d, there is a unique $\hat{r} > 0$, $k + d + \hat{r} \leq n - 3$, s.t. $\Delta\sigma^{k,d,r} > 0$ for all $r \leq \hat{r}$ and $\Delta\sigma^{k,d,r} < 0$ for all $r > \hat{r}$. Also, if $\Delta\sigma^{k,d,r} \geq 0$, then $\Delta\sigma^{k,d+1,r} > 0$.

Proof of Claim 2. Let $s = k + d$ and

$$q := E(\theta|s, r) = \int_0^1 \theta^{s+1}(1-\theta)^r \frac{(r+s+1)!}{r!s!} d\theta = \frac{s+1}{s+r+2} \tag{26}$$

Then,

$$\pi(\sigma_1^{k,d,r}) = \sum_{j=j^*-d}^{n-(s+r+1)} Pr(j|s+1, r), \tag{27}$$

$$\pi(\sigma_2^{k,d,r}) = q \sum_{j=j^*-d-1}^{n-(s+r+1)} Pr(j|s+1,r) + (1-q) \sum_{j=j^*-d}^{n-(s+r+2)} Pr(j|s+1,r+1). \tag{28}$$

$$\Delta\sigma^{k,d,r} \equiv \pi(\sigma_1^{d,r}) - \pi(\sigma_2^{d,r}) = -qPr(j=j^*-d-1|s+1,r)+ \\ (1-q)\left[\sum_{j=j^*-d}^{n-(s+r+1)} Pr(j|s+1,r) - \sum_{j=j^*-d}^{n-(s+r+2)} Pr(j|s+1,r+1)\right]. \tag{29}$$

Applying (22) yields:

$$Pr(j|s+1,r) - Pr(j|s+1,r+1) = \frac{(j+s+1)!(r+s+2)!(n-(j+s+1))!}{(n+1)!(r+1)!(s+1)!} \times \\ \left((r+1)\binom{n-(r+s+1)}{j} - (r+s+3)\binom{n-(r+s+2)}{j}\right). \tag{30}$$

Using (30) and the fact that $Pr(j=n-(s+r+1)|s+1,r+1)=0$, we can rewrite the second term in (29) as follows:

$$\sum_{j=j^*-d}^{n-(s+r+1)} Pr(j|s+1,r) - \sum_{j=j^*-d}^{n-(s+r+2)} Pr(j|s+1,r+1) = \sum_{j=j^*-d}^{n-(s+r+1)} (Pr(j|s+1,r) - Pr(j|s+1,r+1)) = \\ \frac{(j^*-d)(r+s+2)!(j^*-d+s+1)!(d-j^*+n-s)!((r+s+3)\binom{n-r-s-2}{j^*-d} - (r+1)\binom{n-r-s-1}{j^*-d})}{(n+1)!(r+1)!(s+1)!(d(r+s+3)-j^*(r+s+3)+(s+2)(n-r-s-1))}. \tag{31}$$

Using (22) we can also compute:

$$-qPr(j=j^*-d-1|s+1,r) = -\frac{(r+s+1)!(j^*-d+s)!(d-j^*+n-s)!\binom{n-r-s-1}{j^*-d-1}}{(n+1)!r!s!}. \tag{32}$$

Substituting (31) and (32) into (29) yields:

$$\Delta\sigma^{k,d,r} = (s+1-(j^*-d)(j^*-d-n+r+2s+1))\cdot \\ \frac{(r+s+1)!(j^*-d+s)!(n-(r+s+2))!(n+d-j^*-s)!}{(n+1)!r!(s+1)!(j^*-d-1)!(n+d-j^*-r-s)!}. \tag{33}$$

By (33), $\Delta\sigma^{k,d,r} \geq 0$ if and only if

$$k+d+1-(j^*-d)(j^*-d-n+r+2(k+d)+1) \geq 0. \tag{34}$$

Note that the left-hand side of (34) is strictly increasing in d. So, if $\Delta\sigma^{k,d,r} \geq 0$, then $\Delta\sigma^{k,d+1,r} > 0$, as claimed.

Since $j^*-d>0$, the left-hand side of (34) is decreasing in k and r. So, to complete Step 1 it is enough to show that (34) holds at $r=1$ and the maximal possible k equal to k^*-1-d

(recall that by assumption $k+d<k^*$). In this case, (34) becomes:

$$k^* - (j^* - d)(j^* - d - n + 2k^*) \geq 0. \tag{35}$$

The inequality (35) holds strictly since $j^* + 2k^* \leq n-1$ by (16). This completes the proof of Claim 2.

Claim 3. *Consider some period $t \in \{1, ..., k^*\}$ and suppose that the state is $k = t-1, d = 0, r = 0$ (i.e., the agent has invested in all previous periods). Then it is optimal to invest at t.*

Note that $k^* \leq \frac{n-1}{2}$ for odd n and $k^* \leq \frac{n}{2} - 1$ for even n. So, $t \leq k^* \leq n-2$ for all $n \geq 3$, and hence the strategies $\sigma_i^{(k,d,r)}$, $i \in \{1, 2\}$ are feasible at all $t \in \{1, ..., k^*\}$, and Claim 2 applies.

The rest of the proof is by contradiction. So suppose that it is optimal for the agent to acquire a signal at t.

Then there is t', $t < t' < n$, and a history $h'(t')$ -in which the agent invests at any $t \in \{0, ..., t-1\}$, acquires a signal at t and allocates the budget optimally at $s \in \{t, ..., t'-1\}$ - such that it is optimal to invest at t'.

Such t' exists because if it was optimal for the agent to acquire a signal at any $s \geq t$, when the state at t is $k = t-1, d = 0, r = 0$, then this would contradict the optimality of investing k^* in the static case.

Furthermore, we can choose t' and the history $h(t')$ s.t. $t' < n$ and at any s, $s > t'$, it is optimal to acquire a signal. Such t' exists because it is optimal to acquire a signal at $s = n$.

In the sequel we will establish that the agent can increase her payoff by using a different strategy that involves reallocation of the investment at t' to an earlier period, implying that it cannot be optimal to acquire a signal at t.

There are four possible cases to consider:

(a) the agent acquires a signal at $t'-1$ and invests at t' no matter what the realization of the signal acquired at $t'-1$;

(b) the agent acquires a signal at $t'-1$ and invests at t' if this signal is a failure, and acquires a signal at t' if this signal is a success;

(c) the agent acquires a signal at $t'-1$ and invests at t' if this signal is a success, and acquires a signal at t' if this signal is a failure;

(d) the agent invests at $t'-1$ and at t';

Below, we establish that in cases (a)-(d) the agent has a profitable deviation from the candidate optimal strategy.

Case a.

The optimal continuation strategy in case (a) requires the agent to acquire a signal at $t'-1$, to invest at t', and acquire a signal at any s, $s > t$. This strategy is not optimal because it is dominated by the following continuation strategy: invest at $t'-1$ and acquire a signal at any $s \in \{t', ..., n\}$. This claim is true since under both continuation strategies a signal acquired at s, $s > t'$ has the same probability of success. However, the signal acquired at time period

t' following the alternative continuation strategy has a higher probability of success than the signal acquired at time period $t'-1$ according to the original continuation strategy. So, the probability of approval is higher under the alternative strategy.

Case b.

In this case, the agent's optimal continuation strategy starting at $t'-1$ is to acquire a signal at $t'-1$; at t' to invest if the signal acquires at $t'-1$ is a failure and to acquire a signal if the last signal is a success; to acquire a signal at any s, $s \in \{t'+1, ..., n\}$. Therefore, the continuation strategy $\sigma_1^{k',d',r'}$ is optimal for the agent at t' and, in particular, dominates the strategy $\sigma_2^{k',d',r'}$. So, $\Delta\sigma^{k',d',r'} \geq 0$. As shown in Step 1 of Claim 2, $\Delta\sigma^{k',d',r'} \geq 0$ implies that $\Delta\sigma^{k',d'+1,r'-1} > 0$. So continuation strategy σ_1 strictly dominates the strategy σ_2 at $t'-1$. But in our case (b) the agent optimally follows the continuation strategy σ_2 at $t'-1$. A contradiction.

Case c.

In this case, if the signal acquired at $t'-1$ is a success, then the optimal continuation strategy is to invest at t' and acquire a signal at any $s \in \{t'+1, ..., n\}$. So, comparing strategies $\sigma_I^{t'}$ and $\sigma_S^{t'}$, $\sigma_I^{t'}$ dominates $\sigma_S^{t'}$ at t' after a successful signal acquired at $t'-1$. Hence, the opposite of the inequality (25) holds at t' after a successful signal at t' i.e., when the state is (k', d', r'). So, we must have $2(k'-k^*)+d'+r'+2 \leq 0$.

Now, suppose that the signal at $t'-1$ is a failure. Then the state at t' is $(k', d'-1, r'+1)$, and so the following inequality $(2(k'-k^*)+d'+r'+2 \leq 0)$ still holds. Therefore, at t' after a successful signal acquired at $t'-1$, $\sigma_I^{t'}$ dominates $\sigma_S^{t'}$. Therefore, the continuation strategy of investing at t' and acquiring signals at any s, $s \in \{t'+1, ..., n\}$, is optimal after any realization of the signal at $t'-1$. But this strategy is strictly dominated by the strategy of investing at $t'-1$ and acquiring signals at any s, $s \in \{t', ..., n\}$. So, the original strategy cannot be optimal.

Case d.

Let $\hat{t} \in \{k^*-w, ..., t'-1\}$ be such that in history $h'(t')$ the agent acquires a signal at $\hat{t}$ and invest at any $t \in \{\hat{t}+1, ..., t'\}$.

First, suppose that the agent's strategy prescribes investing at $\hat{t}+1$ after any realization of a signal at $\hat{t}$ and subhistory $h'(t'; \hat{t}-1)$ from time period 0 to time period $\hat{t}-1$. Then this strategy cannot be optimal because the agent can strictly increase her payoff by following an alternative strategy that coincides with the original one except at $\hat{t}$ and $\hat{t}+1$. The alternative strategy prescribes to invest at $\hat{t}$ after subhistory $h'(t', \hat{t})$; then to acquire a signal at $\hat{t}+1$.

Now suppose that the agent's strategy prescribes to invest at $\hat{t}+1$ after one realization of a signal at $\hat{t}$ and subhistory $h'(t'; \hat{t}-1)$, and to acquire a signal after the other realization of the signal and subhistory $h'(t'; \hat{t}-1)$. Particularly, let us consider the case when the optimal strategy prescribes to invest after a failed signal at $\hat{t}$ and subhistory $h'(t'; \hat{t}-1)$, and prescribes to acquire a signal after a successful signal at $\hat{t}$ and subhistory $h'(t'; \hat{t}-1)$. The proof in the other case is the same.

First, suppose that the signal acquired at $\hat{t}$ fails. Then the optimal strategy induces history $h'(t')$. Let (k', d', r') be the state at time t' corresponding to the history $h'(t')$. Since it is optimal to invest at t' and to acquire a signal at any period t, $t > t'$, strategy $\sigma_I^{t'}$ is more profitable at t' than $\sigma_S^{t'}$, and strategy $\sigma_S^{t'+1}$ is more profitable at $t'+1$ than $\sigma_I^{t'+1}$. By inequality (25) established in Claim 1, this implies that $2(k'-k^*)+d'+r'+2 \leq 0 \leq 2(k'+1-k^*)+d'+r'+2$.

Now suppose that the signal acquired at $\hat{t}$ is a success and so the agent acquires another signal at $\hat{t}+1$. If the optimal strategy is such that in the sequel the agent always invests in some period t, $\hat{t}+1 < t \leq t'$, the agent can improve his payoff by following an alternative strategy which prescribes to invest at $\hat{t}$ and acquire a signal at t and otherwise to follow the same strategy. So, the optimal strategy with a positive probability induces a history h'' s.t. the investment k'' made by $t'+1$ in h'' is less than investment in h by at least 2 units. So consider the state of the world (k'', d'', r'') at time $t'+1$ after history h''. Then $k''+d''+r'' = k'+1+d'+r'$ and $k'' \leq k'-1$. Therefore, $2(k''-k^*)+d''+r''+2 \leq 2(k'-k^*)+d'+r'+2 \leq 0$. It follows that after history h'' it is optimal to invest at $t'+1$. But this contradicts the definition of t' as the latest investment period when the agent makes an investment. This completes the proof of Claim 3.

The claim of the Theorem now directly follows from Claims 1 and 3.

Q.E.D.

Proof of Proposition 2: First, let us establish which strategies cannot be supported in an equilibrium:

Claim 1: There is no equilibrium with $j^* = 0$ and $k^* > 0$.

Suppose to the contrary that there exist an equilibrium in which the agent chooses $k^* > 0$ and obtains r^* signals and the principal uses threshold $j^* = 0$. Then the agent has a profitable deviation: choose $k = 0$ and obtain any number of signals to save the cost $b > 0$.

Claim 1 implies that there is no equilibrium with $k^* = n$. Using Claim 1, we can now prove the following claim:

Claim 2: If $j^* \geq 1$, the strategy of investing $k' > 0$ and obtaining $r' > 0$ signals s.t. $k'+r' < n$ is dominated by a strategy of investing $n-r'$ and obtaining r' signals.

The proof is obvious since investing $n-r' > k'$ instead of k' increases the probability that any given signal is a success and hence raises the probability of the project approval.

Claims 1 and 2 imply that, if there exists an equilibrium with a positive investment $k' > 0$, then in this equilibrium the agent obtains $n-k'$ signals.

Next, to complete the proof that in equilibrium the agent invests $k^* = \lceil c(n+2) \rceil - 2$ and commits to disclose $n-k^*$ signals, while the principal uses approval threshold $j^* = 1$, we rule out two types of deviations: (i) investing zero; (ii) investing $k' > 0$, $k \neq k^*$.

Claim 3: Deviation to $k = 0$ from $k^* = \lceil c(n+2) \rceil - 2$ is unprofitable for the agent.

Suppose that the agent deviates to $k = 0$ and r signals, which are then disclosed. Given the principal's approval threshold $j' \leq n$, after this disclosure and actual investment level $k = 0$, the principal approves the project with probability $1 - \frac{j'}{r+1} \leq 1 - \frac{j'}{n+1}$.

In the candidate equilibrium, $j^* = 1$ and the probability of persuading the principal is $1 - \frac{(k^*+1)!(n-k^*)!}{(n+1)!}$. So to establish the claim it is sufficient to show that the following inequality holds:

$$\frac{1}{n+1} - \frac{(k^*+1)!(n-k^*)!}{(n+1)!} \geq \frac{n-2}{n(n+1)}. \tag{36}$$

Now, consider the function $\frac{(k^*+1)!(n-k^*)!}{(n+1)!}$ for $k^* \in \{1, .., n-2\}$ (recall that $k^* \leq n-2$). It it easy to check that it reaches a maximum at the corners i.e., at $k^* = 1$ and at $k^* = n-2$ where it takes the value $\frac{2}{n(n+1)}$. Indeed, this follows immediately from the fact that $\Gamma(x^*+2)\Gamma(n-x^*+1)$ is a convex function ox x on $[1, n-2]$. Using either $k^* = 1$ or $k^* = n-2$ in (36) yields

$$\frac{1}{n+1} - \frac{2}{n(n+1)} = \frac{n-2}{n(n+1)}.$$

So, (36) holds, and hence the agent has no incentive to deviate to zero investment.

Claim 4: A deviation to $k' \neq k^* = \lceil c(n+2) \rceil - 2$ is unprofitable for the agent.

Note that by Claim 2 we only need to consider deviations that involve investment k' and $n-k'$ signals, following the disclosure of which the principal uses her optimal decision rule and approves the project only if the number of successes j is such that $j \geq \max\{\lceil c(n+2) \rceil - k' - 1, 1\}$.

First, suppose that $k' \geq \lceil c(n+2) \rceil - 1$ and so $j = 1$. Since $c > \frac{1}{2}$, it follows that $k' > \frac{n}{2}$. Then, given the approval threshold $j = 1$, the probability that the project is approved is equal to:

$$1 - \frac{(k'+1)!(n-k')!}{(n+1)!}. \tag{37}$$

Consider now the agent deviating and choosing investment $k'' = k' - 1$. In this case, $k'' \geq \lceil c(n+2) \rceil - 2$, so the principal's approval threshold remains $j = 1$, and so the probability of the project approval is equal to $1 - \frac{k'!(n-k'+1)!}{(n+1)!}$. This probability is greater than the value of (37) because $k'!(n-k'+1)! < (k'+1)!(n-k')!$ The latter inequality holds because it is equivalent to $n - k' + 1 < k' + 1$ which holds because $k' > \frac{n}{2}$.

Thus, it follows that the agent is strictly better off choosing $k' = \lceil c(n+2) \rceil - 2$ than any larger investment.

Next, let us show that the agent would not deviate to $k'' < \lceil c(n+2) \rceil - 2$. We prove this by showing that, when $k \leq \lceil c(n+2) \rceil - 2$, the agent's payoff when she chooses k is greater than the payoff that she gets by choosing $k-1$. First, when $k \leq \lceil c(n+2) \rceil - 2$ the principal uses threshold $j(k)$ s.t. $j(k) + k = \lceil c(n+2) \rceil - 1$. This implies that the agent's payoff when she chooses k is greater than the payoff that she gets by choosing $k-1$ iff

$$\frac{(n-k)!}{(j(k)-1)!} < \frac{(n-k+1)!}{j(k)!}.$$

The latter inequality is equivalent to $n + 1 > j(k) + k$, which holds because $j(k) + k = \lceil c(n+2) \rceil - 1 < n + 1$. The latter inequality holds because $c < \frac{n+1}{n+2}$. *Q.E.D.*

Proof of Proposition 3: Let j^c denote the approval threshold that the principal's commits to. Let us first characterize the agent's best response to $j^c \in \{0, n\}$. The optimality conditions (16) in the proof of Proposition 1 implies that for $c \in [\frac{1}{2}, \frac{n+1}{n+2}]$. the agent's best response to threshold $j^c \in \{1, ..., n-2\}$ is to invest $k(j^c)$ s.t. $\frac{n-j^c-1}{2} \leq k(j^c) \leq \frac{n-j^c+1}{2}$, and acquire $n-k(j^c)$ signals. This inequality implies that $k(j^c) = \frac{n-j^c}{2}$ if $n - j^c$ is even, and $k(j^c) \in \{\frac{n-j^c-1}{2}, \frac{n-j^c+1}{2}\}$ if $n - j^c$ is odd. In the latter case we choose the higher investment $k(j^c) = \frac{n-j^c+1}{2}$.

Note that the agent does not wish to deviate from $k(j^c) > 0$ to $k = 0$ when $j^c \in \{1, ..., n-2\}$ and Assumption 1 holds i.e., $b \leq \frac{n-2}{n(n+1)}$. To establish this, let us show that the agent prefers action pair ($k = 1$, $r = n - 1$) to action pair ($k = 0$, $r = n$) under threshold $j^c \leq n - 2$, which then implies that she prefers $k(j^c)$ to $k = 0$. Indeed, by (10), the probability of at least j successes under ($k = 0$, $r = n$) is $Pr(j' \geq j|k = 0, r = n) = 1 - \frac{j}{n+1}$, and the probability of obtaining at least j successes under ($k = 1, r = n-1$) is $Pr(j' \geq j|k = 1, r = n-1) = 1-\frac{(j+1)j}{(n+1)n}$. Then,

$$Pr(j' \geq j|k = 1, r = n - 1) - Pr(j' \geq j|k = 0, r = n) = \frac{j(n - j - 1)}{n(n + 1)}. \tag{38}$$

When $n \geq 3$, then on the domain $j \in \{1, ..., n - 2\}$, (38) reaches a minimum $\frac{n-2}{n(n+1)}$ both at $j = 1$ and at $j = n - 2$. Thus, if $b < \frac{n-2}{n(n+1)}$, the agent prefers investment $k = 1$ to $k = 0$, and so a priori she would not deviate from $k(j^c) > 0$ to $k = 0$. If $j^c = n - 1$, then $Pr(j' \geq n - 1|k = 1, r = n - 1) = Pr(j' = n - 1|k = 1, r = n) = Pr(j' \geq n - 1|k = 0, r = n) = \frac{2}{(n+1)}$, and the agent's best response is $k(j^c = n - 1) = 0$. If $j^c = n - 1$, then naturally the agent's best response is $k(n) = 0$.

Now, let us consider the principal's optimal choice of the threshold j^c. Let $\pi(k, j)$ be the principal's expected payoff under threshold j when the agent invests k and acquires $n - k$ signals. We have:

$$\pi(k, j) = \sum_{j'=j}^{n-k} Pr(j'|k, n)\Big(\frac{k + j' + 1}{n + 2} - c\Big) = \sum_{j'=j}^{n-k} \frac{(k + 1)(j + k)!(n - k)!}{j!(n + 1)!}\Big(\frac{k + j' + 1}{n + 2} - c\Big) \tag{39}$$

Recall that $Pr(j|k, n) = \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}$ is the probability of j successes with $n - k$ signals and investment k.

First, let us deal with special boundary cases, $j^c = n - 1$, $j^c = n$ and $j^c = 0$. Consider $j^c = n - 1$. Since $Pr(j' = n - 1|k = 0, r = n) = Pr(j' = n|k = 0, r = n) = \frac{1}{(n+1)}$, $\pi(k(n - 1) = 0, j^c = n - 1) = \frac{(\frac{n}{n+2}-c)+(\frac{n+1}{n+2}-c)}{n+1}$. Also, $\pi(k = 1, j^c = n - 1) = 2\frac{\frac{n+1}{n+2}-c}{n+1}$ and $\pi(k(n - 2) = 1, j^c = n-2) = Pr(j = n-2|k = 1, r = n-1)\left(\frac{n}{n+2} - c\right) + Pr(j = n-1|k = 1, r = n-1)\left(\frac{n+1}{n+2} - c\right) = \frac{2(n-1)}{n(n+1)}\left(\frac{n}{n+2} - c\right) + \frac{2}{n+1}\left(\frac{n+1}{n+2} - c\right)$. Thus, $\pi(k(n-2) = 1, j^c = n-2) > \pi(k(n-1) = 0, j^c = n-1)$, so $j^c = n - 1$ is not an optimal choice for the principal when $c \leq \frac{n}{n+2}$. In the complementary case, when $c > \frac{n}{n+2}$ and also $n \geq 3$, $\pi(k = 1, j^c = n - 1) > \pi(k(n - 2) = 1, j^c = n - 2)$. In this case, the derivation in (46) and below shows that $\pi(k=1, j^c = n-1) < \pi(k(n-3), j^c = n-3)$, so $j^c = n-1$ is not optimal.

Next, consider $j^c = n$. We have $\pi(k(n) = 0, j^c = n) = \frac{\frac{n+1}{n+2} - c}{n+1} = \frac{1}{2}\pi(k=1, j^c = n-1) > 0$. Since (46) and the derivations below establish that $\pi(k=1, j^c = n-1) < \pi(k(n-2), j^c = n-2)$ when $n-1$ is even and $n \geq 3$ and $\pi(k=1, j^c = n-1) < \pi(k(n-3), j^c = n-3)$ when $n-1$ is odd and so $n \geq 4$, we conclude that $j^c = n$ is not optimal when $n \geq 3$.[14]

Now, consider $j^c = 0$. We have $k(0) = 0$, so with $r = n$ and $c \in [\frac{1}{2}, \frac{n+1}{n+2}]$, we have $\pi(0,0) = \frac{1}{n+1}\sum_{j=0}^{n}\left(\frac{j+1}{n+2} - c\right) = \frac{1}{2} - c < \frac{\frac{n+1}{n+2} - c}{n+1} = \pi(k=0, j=n)$. Thus, $j^c = 0$ is not optimal.

Next, let us show that $\pi(k(j^c-1), j^c-1) - \pi(k(j^c), j^c) > 0$ for all $j^c \in \{2, ..., n\}$ when $n - j^c$ is even. Since in this case $k(j^c) = \frac{n-j^c}{2}$ and $k(j^c - 1) = \frac{n-j^c}{2} + 1$, we have:

$$
\begin{aligned}
&\pi(k(j^c-1), j^c-1) - \pi(k(j^c), j^c) = \sum_{j=j^c-1}^{n-\frac{n-j^c}{2}-1} Pr(j|\frac{n-j^c}{2}+1, n)\Big(\frac{\frac{n-j^c}{2}+1+j+1}{n+2} - c\Big) \\
&- \sum_{j=j^c}^{n-\frac{n-j^c}{2}} Pr(j|\frac{n-j^c}{2}, n)\Big(\frac{\frac{n-j^c}{2}+j+1}{n+2} - c\Big) \\
&= \Big(\frac{\frac{n+j^c}{2}+1}{n+2} - c\Big)\sum_{j=j^c}^{n-\frac{n-j^c}{2}} (Pr(j-1|\frac{n-j^c}{2}+1, n) - Pr(j|\frac{n-j^c}{2}, n)) \\
&+ \frac{1}{n+2}\sum_{j'=j^c+1}^{n-\frac{n-j^c}{2}}\sum_{j=j'}^{n-\frac{n-j^c}{2}} (Pr(j-1|\frac{n-j^c}{2}+1, n) - Pr(j|\frac{n-j^c}{2}, n)) = \\
&\Big(\frac{\frac{n+j^c}{2}+1}{n+2} - c\Big)\left(\frac{(\frac{n+j^c}{2})!(\frac{n+j^c}{2})!}{(n+1)!(j^c-1)!} - \frac{(\frac{n+j^c}{2})!(\frac{n+j^c}{2}-1)!}{(n+1)!(j^c-2)!}\right) \\
&+ \frac{1}{n+2}\sum_{j'=j^c+1}^{\frac{n+j^c}{2}}\left(\frac{(\frac{n-j^c}{2}+j')!(n-\frac{n-j^c}{2})!}{(n+1)!(j'-1)!} - \frac{(\frac{n-j^c}{2}+j')!(n-\frac{n-j^c}{2}-1)!}{(n+1)!(j'-2)!}\right) = \\
&\Big(\frac{\frac{n+j^c}{2}+1}{n+2} - c\Big)\frac{(\frac{n+j^c}{2})!(\frac{n+j^c}{2}-1)!}{(n+1)!(j^c-1)!}\left(\frac{n-j^c}{2}+1\right) + \\
&\frac{1}{n+2}\sum_{j'=j^c+1}^{\frac{n+j^c}{2}}\frac{(\frac{n-j^c}{2}+j')!(\frac{n+j^c}{2}-1)!}{(n+1)!(j'-1)!}\left(\frac{n+j^c}{2} - j' + 1\right) = \\
&\frac{(\frac{n+j^c}{2}-1)!}{(n+2)!}\left(\Big(\frac{n+j^c}{2}+1-c(n+2)\Big)\frac{(\frac{n+j^c}{2})!}{(j^c-1)!}\left(\frac{n-j^c}{2}+1\right) + \sum_{j'=j^c+1}^{\frac{n+j^c}{2}}\frac{(\frac{n-j^c}{2}+j')!}{(j'-1)!}\left(\frac{n+j^c}{2}-j'+1\right)\right)
\end{aligned}
\tag{40}
$$

where the first equality holds by (39), the third equality holds by (11) in Lemma (5), and the second, forth and fifth equalities hold by rearrangement.

[14] When $n = 2$, (38) implies that the agent's best response to $j^c = 1$ is $k = 0$. The agent's best response to $j^c = 2$ is $k = 0$ also. But since $\pi(k=0, j^c = 1; n = 2) = \frac{(\frac{1}{2}-c)+(\frac{3}{4}-c)}{3} < \frac{\frac{3}{4}-c}{3} = \pi(k=0, j^c = 2; n = 2)$, it is optimal for the principal to set $j^c = 2$, with $k(2) = 0$.

To show that (40) is positive, let us consider the terms in brackets in the last line of (40). Consider the first term. Since $c \leq \frac{n+1}{n+2}$, we have:

$$\left(\frac{n+j^c}{2}+1-c(n+2)\right)\frac{(\frac{n+j^c}{2})!}{(j^c-1)!}\left(\frac{n-j^c}{2}+1\right) \geq \frac{n-j^c}{2}\frac{(\frac{n+j^c}{2})!\left(\frac{n-j^c}{2}+1\right)}{(j^c-1)!} \tag{41}$$

Next, let us simplify the second term in brackets on the last line of (40). We have:

$$\sum_{j'=j^c+1}^{\frac{n+j^c}{2}} \frac{(\frac{n-j^c}{2}+j')!}{(j'-1)!}\left(\frac{n+j^c}{2}-j'+1\right) = \frac{n+j^c}{2}\sum_{j=j^c+1}^{\frac{n+j^c}{2}} \frac{(\frac{n-j^c}{2}+j')!}{(j'-1)!} - \sum_{j'=j^c+1}^{\frac{n+j^c}{2}} \frac{(\frac{n-j^c}{2}+j')!}{(j'-2)!} =$$

$$\frac{n+j^c}{2}\sum_{j=j^c}^{\frac{n+j^c}{2}-1} \frac{(\frac{n-j^c}{2}+1+j)!}{j!} - \sum_{j=j^c-1}^{\frac{n+j^c}{2}-2} \frac{(\frac{n-j^c}{2}+2+j)!}{j!} =$$

$$\frac{n+j^c}{2}\left(\sum_{j=0}^{\frac{n+j^c}{2}-1} \frac{(\frac{n-j^c}{2}+1+j)!}{j!} - \sum_{j=0}^{j^c-1} \frac{(\frac{n-j^c}{2}+1+j)!}{j!}\right)$$

$$-\sum_{j=0}^{\frac{n+j^c}{2}-2} \frac{(\frac{n-j^c}{2}+2+j)!}{j!} + \sum_{j=0}^{j=j^c-2} \frac{(\frac{n-j^c}{2}+2+j)!}{j!}$$

$$= \left(\frac{n+j^c}{2}\frac{(n+1)!}{(\frac{n-j^c}{2}+2)(\frac{n+j^c}{2}-1)!} - \frac{(n+1)!}{(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-2)!}\right)$$

$$-\left(\frac{n+j^c}{2}\frac{(\frac{n+j^c}{2}+1)!}{(\frac{n-j^c}{2}+2)(j^c-1)!} - \frac{(\frac{n+j^c}{2}+1)!}{(\frac{n-j^c}{2}+3)(j^c-2)!}\right) =$$

$$= \frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!} - \frac{(\frac{n+j^c}{2}+1)!\left((\frac{n-j^c}{2}+1)(\frac{n-j^c}{2}+2)+\frac{n+j^c}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} \tag{42}$$

Using (41) and (42) in (40), yields:

$$\pi(k(j^c-1),j^c-1)-\pi(k(j^c),j^c) = \frac{(\frac{n+j^c}{2}-1)!}{(n+2)!}\left(\frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!} - \right.$$

$$\left.\frac{(\frac{n+j^c}{2}+1)!\left((\frac{n-j^c}{2}+1)(\frac{n-j^c}{2}+2)+\frac{n+j^c}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} - \frac{(\frac{n+j^c}{2})!\left(\frac{n-j^c}{2}+1\right)}{(j^c-1)!}\frac{n-j^c}{2}\right) \tag{43}$$

The last two terms (43) can be simplified to yield:

$$\frac{(\frac{n+j^c}{2}+1)!\left((\frac{n-j^c}{2}+1)(\frac{n-j^c}{2}+2)+\frac{n+j^c}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} + \frac{(\frac{n+j^c}{2})!\left(\frac{n-j^c}{2}+1\right)}{(j^c-1)!}\frac{n-j^c}{2} =$$

$$= \frac{(\frac{n+j^c}{2}+1)!\left((\frac{n-j^c}{2}+1)(\frac{n-j^c}{2}+2)+\frac{n+j^c}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} + \frac{(\frac{n+j^c}{2})!\left(\frac{n-j^c}{2}+1\right)(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)\frac{n-j^c}{2}}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!}$$

$$= \frac{(\frac{n+j^c}{2})!\left(2n-j^c+1+\left(\frac{n-j^c}{2}\right)^2+\frac{n+j^c}{2}\left(\frac{n+j^c}{2}+1\right)\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} = \frac{(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} \tag{44}$$

Substituting (44) into (43) we finally obtain:

$$\pi(k(j^c-1), j^c-1) - \pi(k(j^c), j^c) =$$
$$\frac{(\frac{n+j^c}{2}-1)!}{(n+2)!}\left(\frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!} - \frac{(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!}\right) =$$
$$\frac{(\frac{n+j^c}{2}-1)!}{(n+2)!(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!}\left((n+2)! - \frac{(\frac{n+j^c}{2}-1)!(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}\right)}{(j^c-1)!}\right) > 0 \tag{45}$$

To ascertain the inequality on the last line of (45), note that the expression in brackets on its last line is positive when $j^c = n$ (which is established by inspection) and decreases in j^c.

Next, suppose that $n - j^c$ is odd. Then $k(j^c) = \frac{n-j^c+1}{2}$ and $k(j^c-1) = \frac{n-j^c+1}{2}$ (note that as explained above, here we can take by convention $k(n-1) = 1$). So we have:

$$\pi(k(j^c-1), j^c-1) - \pi(k(j^c), j^c) = \sum_{j=j^c-1}^{n-\frac{n-j^c+1}{2}} Pr(j|\frac{n-j^c+1}{2}, n)\Big(\frac{\frac{n-j^c+1}{2}+j+1}{n+2} - c\Big)$$
$$- \sum_{j=j^c}^{n-\frac{n-j^c+1}{2}} Pr(j|\frac{n-j^c+1}{2}, n)\Big(\frac{\frac{n-j^c+1}{2}+j+1}{n+2} - c\Big) = Pr(j^c-1|\frac{n-j^c+1}{2}, n)\Big(\frac{\frac{n+j^c+1}{2}}{n+2} - c\Big)$$
$$= \frac{(\frac{n-j^c+1}{2}+1)(\frac{n+j^c-1}{2})!(\frac{n+j^c-1}{2})!}{(j^c-1)!(n+1)!}\Big(\frac{\frac{n+j^c+1}{2}}{n+2} - c\Big). \tag{46}$$

where the first equality holds by (39), the second equality holds by rearrangement, and the third equality holds by (11) in Lemma 5.

Combining (45) and (46) we obtain that for even $n - j^c$ we have:

$$\pi(k(j^c-1), j^c-1) - \pi(k(j^c+1), j^c+1) =$$
$$\frac{(\frac{n+j^c}{2}-1)!}{(n+2)!}\left(\frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!} - \frac{(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!}\right)$$
$$+ \frac{(\frac{n-j^c}{2}+1)(\frac{n+j^c}{2})!(\frac{n+j^c}{2})!}{(j^c)!(n+1)!}\Big(\frac{\frac{n+j^c+2}{2}}{n+2} - c\Big). \tag{47}$$

Factoring out $\frac{(\frac{n+j^c}{2}-1)!}{(n+2)!}$ from (47) yields: $\frac{(\pi(k(j^c-1),j^c-1)-\pi(k(j^c+1),j^c+1))(n+2)!}{(\frac{n+j^c}{2}-1)!} =$

$$\frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!} - \frac{(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!} - \frac{(\frac{n-j^c}{2}+1)(\frac{n+j^c}{2})!}{(j^c)!}\frac{(n-j^c)}{2} =$$
$$\frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!}$$

$$
\begin{aligned}
&-\frac{(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!}-\frac{(\frac{n-j^c}{2}+1)(\frac{n+j^c}{2})!(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c)!}\frac{(n-j^c)}{2}\\
&=\frac{(n+2)!}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(\frac{n+j^c}{2}-1)!}\\
&-\frac{(\frac{n+j^c}{2})!\left(\frac{5n-j^c}{2}+1+\frac{n^2+(j^c)^2}{2}+(\frac{n-j^c}{2}+1)(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)\frac{(n-j^c)}{2j}\right)}{(\frac{n-j^c}{2}+2)(\frac{n-j^c}{2}+3)(j^c-1)!}>0 \qquad (48)
\end{aligned}
$$

Simple inspection establishes that this expression has a positive sign when $n \geq 5$ and any $j^c \in \{2, n-1\}$ and $n = 4$ and $j^c = 2$ (recall that we also require $n - j$ to be even here).

So, in combination, (45) and (48) imply that , when $n \geq 3$ and n is even, $\pi(k(1), 1) > \pi(k(j^c), j^c)$ for all $j^c \geq 2$, so $j^c = 1$ is the unique optimal choice for the principal.

When n is odd and $n \geq 5$, then (45) and (48) imply that $\pi(k(2), j^c = 2) > \pi(k(j^c), j^c)$ for all $j^c > 2$. At the same time, (46) implies that $\pi(k(2), 2) > \pi(k(1), 1)$ since $c \geq \frac{n+3}{2(n+2)}$. So, $j^c = 2$ is the unique optimal choice for the principal.

When $n = 3$, $\pi(k(2), 2) > \pi(k(1), 1)$, so $j^c = 2$ is optimal. We have also established above that $j^c = 2$ is optimal when $n = 2$. *Q.E.D.*

Proof of Proposition 4: Recall from Proposition 3 that the level of investment under principal's commitment is $k_p \in \{\frac{n}{2}, \frac{n-1}{2}\}$, depending on whether n is even or odd. On the other hand, the level of investment under the agent's commitment is $k_a := \lceil c(n+2) \rceil - 2$. Next, let us compute the players' expected payoffs.

1. First, consider the case under the principal's commitment, with even n. In this case, $j^c = 1$ and $k(j^c) = \frac{n}{2}$. Then, we have:

   $$Pr(j|k(j^c), n) = \frac{\Gamma\left(\frac{n}{2}+2\right)\Gamma\left(j+\frac{n}{2}+1\right)}{\Gamma(j+1)\Gamma(n+2)}$$

   and $E(\theta|k(j^c), n) = \frac{1}{2} + \frac{j}{n+2}$. Therefore the principal's expected payoff is

   $$\sum_{j=1}^{n-k(j^c)} \left(\frac{\Gamma\left(\frac{n}{2}+2\right)\Gamma\left(j+\frac{n}{2}+1\right)}{\Gamma(j+1)\Gamma(n+2)} - c\right) = \frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - c + \frac{n+2}{n+4}. \qquad (49)$$

2. Second, consider the principal's commitment case with odd n. In this case, optimal commitment threshold is $j^c = 2$. Therefore, if the principal commits to $j^c = 1$, her payoff would be lower than with $j^c = 2$ and would be equal to (49).

3. Third, consider the principal's expected payoff under the agent's commitment. Here, in equilibrium we have $j^c = 1$ and $k_a(j^c = 1) = \lceil c(n+2) \rceil - 2$. With $\hat{k} = c(n+2) - 2$, we

have $E(\theta|\hat{k},n) = c + \frac{j-1}{n+2}$ and

$$Pr(j|\hat{k},n) = \frac{(c(n+2)-1)\Gamma(n-c(n+2)+3)\Gamma(j+c(n+2)-1)}{\Gamma(j+1)\Gamma(n+2)}.$$

Therefore, the principal's expected payoff under the agent's commitment is

$$\begin{aligned}
&\sum_{j=1}^{n-\hat{k}} \frac{(c(n+2)-1)\Gamma(n-c(n+2)+3)\Gamma(j+c(n+2)-1)}{\Gamma(j+1)\Gamma(n+2)} \frac{j-1}{n+2} \\
&= -\frac{1}{c(n+2)} + \frac{\Gamma(c(n+2))\Gamma(n-c(n+2)+3)}{\Gamma(n+3)} - c + 1. \qquad (50)
\end{aligned}$$

Now, let us show that the difference (50)-(49) is positive. This difference is equal to:

$$-\frac{1}{c(n+2)} - \frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} + \frac{\Gamma(c(n+2))\Gamma(n-c(n+2)+3)}{\Gamma(n+3)} - \frac{n+2}{n+4} + 1. \qquad (51)$$

Consider the following derivatives:

$$\frac{\partial \; -\frac{1}{c(n+2)} - \frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - \frac{n+2}{n+4} + 1}{\partial c} = \frac{1}{c^2(n+2)} - \frac{2\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} \qquad (52)$$

that itself decreases in c, and the derivative

$$\begin{aligned}
&\frac{\partial \; \frac{\Gamma(c(n+2))\Gamma(n-c(n+2)+3)}{\Gamma(n+3)}}{\partial c} \\
&= \frac{(n+2)\Gamma(c(n+2))\Gamma(n-c(n+2)+3)(\psi(c(n+2)) - \psi(n-c(n+2)+3))}{\Gamma(n+3)} > 0 \qquad (53)
\end{aligned}$$

since $\psi(c(n+2)) > \psi(n-c(n+2)+3)$. We want to show that (52) $>$ (53). To see this, note that the derivative of (53) with respect to c is positive and equal to:

$$\begin{aligned}
&(n+2)^2\Gamma(c(n+2))\Gamma(n-c(n+2)+3)\times \\
&\frac{\left((\psi^{(0)}(c(n+2)) - \psi^{(0)}(n-c(n+2)+3))^2 + \psi^{(1)}(c(n+2)) + \psi^{(1)}(n-c(n+2)+3)\right)}{\Gamma(n+3)} > 0.
\end{aligned}$$

Then, using $c' := \frac{n-1}{n+2}$, the difference (52) $-$ (53) may be computed as

$$\frac{n+2}{(n-1)^2} - \frac{2\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - \frac{6(n+2)\Gamma(n-1)\left(\psi^{(0)}(n-1) + \gamma - \frac{11}{6}\right)}{\Gamma(n+3)} \qquad (54)$$

Note that (54) is positive if and only if:

$$-2(n-1)^2\Gamma\left(\frac{n}{2}+2\right)^2+(n+2)\Gamma(n+3)-6(n+2)(n-1)^2\Gamma(n-1)\left(\psi^{(0)}(n-1)+\gamma^{EM}-\frac{11}{6}\right)>0.$$

where γ^{EM} is the Euler-Masceroni constant.

In the next step, consider $\hat{c}=\frac{n+4}{2(n+2)}$. Then

$$-\frac{1}{\hat{c}(n+2)}-\frac{(2\hat{c}-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)}-\frac{n+2}{n+4}+1=\frac{\Gamma(\hat{c}(n+2))\Gamma(n-\hat{c}(n+2)+3)}{\Gamma(n+3)}$$

and therefore we conclude that (51)≥ 0 is positive. Thus, the principal's expected payoff under the agent's commitment is weakly higher than the principal's payoff under the principal's commitment.

The principal's expected payoff under her commitment is higher than her payoff in the equilibrium of the disclosure game under the baseline scenario, because the principal can always replicate the outcome of the disclosure game under commitment.

Next, consider the agent's expected payoffs under different regimes. We first show that the agent prefers his commitment to full disclosure to his equilibrium payoff in the disclosure game. We maintain the assumption that the agent wants to invest at least one unit, and therefore we need to compare the probabilities of the project approval in the two environments. First, under the commitment to full signal disclosure the probability of the project approval that is, provided that $j^*=1$ and $k^*=\lceil c(n+2)\rceil-2$:

$$1-\frac{\Gamma(n-\lceil c(n+2)\rceil+3)\Gamma(\lceil c(n+2)\rceil)}{\Gamma(n+2)}.$$

In the equilibrium of the disclosure game, provided that $c\in\left[\frac{n+4}{2(n+2)},\frac{n}{n+2}\right]$, the probability of the project approval is:

$$1-\frac{\Gamma(\lceil c(n+2)\rceil-1)\Gamma(\lceil c(n+2)\rceil)}{\Gamma(n+2)\Gamma(-n+2\lceil c(n+2)\rceil-3)}.$$

Then, the agent prefers the commitment to full signal disclosure if

$$\frac{\Gamma(\lceil c(n+2)\rceil-1)}{\Gamma(n-\lceil c(n+2)\rceil+3)\Gamma(-n+2\lceil c(n+2)\rceil-3)}\geq 1. \tag{55}$$

Due to the properties of the ceiling-function, we have:

$$\frac{\Gamma(\lceil c(n+2)\rceil-1)}{\Gamma(n-\lceil c(n+2)\rceil+3)\Gamma(-n+2\lceil c(n+2)\rceil-3)}\geq\frac{\Gamma(c(n+2)-1)}{\Gamma(n-c(n+2)+3)\Gamma(-n+2c(n+2)-2)}. \tag{56}$$

Now, compute the derivative:

$$\frac{\partial \frac{\Gamma(c(n+2)-1)}{\Gamma(n-c(n+2)+3)\Gamma(-n+2c(n+2)-2)}}{\partial c} =$$

$$\frac{(n+2)\Gamma(c(n+2)-1)(-2\psi((2c-1)(n+2))+\psi(n-c(n+2)+3)+\psi(c(n+2)-1))}{\Gamma((2c-1)(n+2))\Gamma(n-c(n+2)+3)},$$

The last expression is positive for small c, and negative otherwise. Thus, the RHS of (56) is, first, increasing and then decreasing on the given cost interval.

First, consider the realization of the RHS of (56) at $c = \frac{n+4}{2(n+2)}$, that is 1. Second, consider the realization of the RHS of (56) at $c = \frac{n-1}{n+2}$, that is $\frac{1}{6}(n-4)(n-3) \geq 1$ for $n \geq 6$. Given that the RHS of (56) increases in c at $c = \frac{n+4}{2(n+2)}$, it must be the case that the inequality (55) is satisfied on $c \in \left[\frac{n+4}{2(n+2)}, \frac{n-1}{n+2}\right]$, and therefore the agent prefers commitment to full signal disclosure to the equilibrium of the disclosure game.

Next, we show that the agent prefers principal's commitment to the outcome of the disclosure game. To obtain this result, consider, first, the agent's expected payoffs under the principal's commitment:

1. Consider n even; $j^c = 1$ and $k^* = \frac{n}{2}$. In this case the agent's expected payoff is (i.e. by omitting the fixed cost of investment it is just the probability of persuading the principal)

$$1 - \frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+4}{2}\right)}{\Gamma(n+2)},$$

2. Consider n odd; $j^c = 2$ and $k^* = \frac{n-1}{2}$. In this case the agent's expected payoff is

$$1 - \frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+5}{2}\right)}{\Gamma(n+2)}.$$

As we know from the previous part of the proof, in the disclosure game the agent's expected payoff is:

$$1 - \frac{\Gamma(\lceil c(n+2)\rceil - 1)\Gamma(\lceil c(n+2)\rceil)}{\Gamma(n+2)\Gamma(2\lceil c(n+2)\rceil - (n+3))} \tag{57}$$

that is (weakly) decreasing in c. At the lower bound of the cost interval, $c = \frac{n+4}{2(n+2)}$, the expected payoff (57) becomes

$$1 - \frac{\Gamma\left(\left\lceil \frac{n}{2}\right\rceil + 1\right)\Gamma\left(\left\lceil \frac{n}{2}\right\rceil + 2\right)}{\Gamma(n+2)\Gamma\left(-n+2\left\lceil \frac{n}{2}\right\rceil + 1\right)}. \tag{58}$$

Consider n even. Then, since $\lceil \frac{n}{2} \rceil = \frac{n}{2}$, (58) can be expressed as

$$1 - \frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+4}{2}\right)}{\Gamma(n+2)\Gamma\left(-n+2\frac{n}{2}+1\right)} = 1 - \frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+4}{2}\right)}{\Gamma(n+2)}$$

which is exactly the same as the payoff under the principal's commitment with n even. Since the agent's payoff in the game decreases in c, the agent prefers principal's commitment to the outcome of the game.

Consider n odd. Then, since $\lceil \frac{n}{2} \rceil = \frac{n+1}{2}$, (58) can be expressed as

$$1 - \frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+5}{2}\right)}{\Gamma(n+2)\Gamma\left(-n+2\frac{n+1}{2}+1\right)} = 1 - \frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+5}{2}\right)}{\Gamma(n+2)}$$

where the latter equality is satisfied since $\Gamma(2) = 1$. The above payoff is the same as the payoff under the principal's commitment with n odd. So, since the agent's payoff in the game decreases in c, the agent prefers principal's commitment to the outcome of the game. *Q.E.D.*

Proof of Corollary 1: Suppose that the agent is committed to full signal disclosure and the principal is committed to some approval threshold j^c. Then the agent's disclosure has no effect on approval of the project. Therefore, the agent's best response budget allocation would be the same as the one characterized in Proposition 3. So the equilibrium outcome would be the same as the one that emerges when only the principal can commit and which is characterized in Proposition 3. But by Proposition 4, the principal prefers the equilibrium outcome under the agent's commitment to the outcome under her own commitment. So, the principal would be better off not to use any commitment when the agent is committed to disclosure.
Q.E.D.

Proof of Proposition 5: Let $(l, j(.))$ denote the principal's strategy, where $l \in \{0, ..., n\}$ and $j(.) : \{0, ..., n\} \mapsto \{0, ..., n+1\}$ and $(r(.), k(.)) : \{0, ..., n\} \mapsto \{0, ..., n\}^2$ s.t. $r(l) \leq l$ and $k(l) + r(l) \leq n$ for all l, denote the agent's strategy, where l is the ceiling on the number of signals set by the principal, $j(l')$ is the principal's approval threshold when she uses the ceiling l', $k(l)$ is the agent's investment and $r(l)$ is the number of signals acquired by the agent when the principal sets the signal ceiling l. By convention, we set $j(l') = n+1$ if the principal never approves the project after setting threshold l'.

The principal's equilibrium strategy $(l^e, j^e(.))$ and the agent's equilibrium strategy $(r^e(.), k^e(.))$ must be sequentially rational. That is, for each $l \in \{0, ..., n\}$, we must have $r^e(l) \leq l$ and the strategies $(r^e(l), k^e(l))$ and $j^e(l)$ must constitute an equilibrium of the continuation game, and l^e must maximize the principal's expected payoff over $l \in \{0, ..., n\}$ given these continuation equilibrium strategies $(r^e(.), k^e(.))$ and $j^e(.)$.

Note that the argument of Lemma 1 implies that the principal's approval strategy $j(.)$ is sequentially rational iff:

$$j(l) = \max\{0, \lceil c(r(l) + k(l) + 2)\rceil - k(l) - 1\}. \tag{59}$$

On the other hand, the sequential rationality of the agent's strategy $(r(.), k(.))$ implies the following. If $l > 0$ and $j(l)$ is such that $0 < j(l) \leq l$, then $r(l) = \min\{l, n - k(l)\}$ i.e., the agent will either acquire the maximum possible number of signals l or, if $n - k(l) < l$, allocate all the budget remaining after investment to signals, for otherwise the agent could get a strictly higher payoff by acquiring more signals. Also, if $0 < j(l) \leq l$, a sequentially rational $k(l)$ is such that either $k(l) = 0$ or $k(l) \geq n - l$. Indeed, if $0 < k(l) < n - l$, then the agent could increase her payoff by increasing her investment level to $n - l$ since she cannot acquire more than l signals. Note that if $k(l) > 0$, then, combining $k(l) \geq n - l$ and $r(l) = \min\{l, n - k(l)\}$ implies that $r(l) + k(l) = n$.

Now, we can proceed to characterize the continuation equilibria for every $l \in \{1, .., n\}$. There are three cases to consider: (1) $l \in \{\lceil c(n + 2)\rceil - 2, ..., n\}$; (2) $l \in \{n + 2 - \lceil c(n + 2), ..., \lceil c(n + 2) - 3\rceil\}$; (3) $l \in \{0, ..., n + 1 - \lceil c(n + 2)\}$.

(1) $l \in \{\lceil c(n+2)\rceil - 2, ..., n\}$. In this case, the equilibrium characterized in Proposition 1 is feasible and gives rise to an equivalent equilibrium in our continuation game, which we adopt here. In this equilibrium $k(l) = (n + 2) - \lceil c(n + 2)\rceil$, $r(l) = n - k(l) = \lceil c(n + 2) - 2$ and $j(l) = 2\lceil c(n + 2)\rceil - (n + 3)$. Note that the equilibrium strategies and outcomes are the same for all l in this range.

(2) $l \in \{n + 2 - \lceil c(n + 2), ..., \lceil c(n + 2)\rceil - 3\}$. Let us show that strategies $k(l) = n - l$, $r(l) = l$, $j(l) = \lceil c(n + 2) + l - n - 1$ constitute a continuation equilibrium in this case.

Note that by (59) $j(l) = \lceil c(n + 2) + l - n - 1$ is a best response to $(k(l) = n - l,\ r(l) = l)$ Let us show that $(k(l) = n - l,\ r(l) = l)$ is a best response to $j(l) = \lceil c(n + 2) + l - n - 1$ for given l. First note that a deviation to $r' > l$ is infeasible. Also, the agent does not wish to deviate to (k', r') s.t. $0 < k < k(l)$, $r' \leq l$ because such deviation involves a lower and positive investment and a lower number of signals. These changes both reduce the probability of the project approval given that we have $j(l) > 0$.

Now let us rule out a deviation to k' s.t. $k' > k(l)$. Since $j(l) > 0$, we should focus on deviations such that $k' \leq n - 1$ and $r' = n - k' \geq 1$. Then by (37), the probability of project approval is equal to $1 - \frac{(k'+j(l))!(n-k')!}{(j(l)-1)!(n+1)!}$. As shown in the proof of Lemma 5, this expression is a concave function of k', is symmetric in k' around $k^m = \frac{n-j(l)}{2}$, reaches a unique maximum at $k^m = \frac{n-j(l)}{2}$, and is decreasing in k' when $k' \geq \frac{n-j(l)}{2}$. But note that $\frac{n-j(l)}{2} = n - \frac{\lceil c(n+2)\rceil+1+l}{2} < n - l = k(l)$ (where the inequality holds because $\lceil c(n + 2) + 1 > \lceil c(n + 2)\rceil - 3 \geq l)$, which implies that a deviation to k', $k' > n - l$, is unprofitable for the agent.

Now let us rule out a deviation to $k' = 0$ while keeping $r' = l$. By (10), the probability of obtaining at least $j \leq r'$ successes under $k = 0$ is $Pr(j' \geq j|k = 0, r') = 1 - \frac{j}{r'+1}$, while the probability of obtaining at least j successes with $k = 1$ and $r' = l$, is $Pr(j' \geq j|k = 1, r') =$

$1-\frac{(j+1)j}{(r'+2)(r'+1)}$. Then,

$$Pr(j' \geq j|k=1,r') - Pr(j' \geq j|k=0,r') = \frac{(r'-j+1)j}{(r'+2)(r'+1)}. \tag{60}$$

When $j \in \{1,...,r'\}$, (60) is nonnegative and reaches its minimum $\frac{r'}{(r'+2)(r'+1)}$ at the corners at $j=1$ or at $j=r'$. Note that $r'=l \leq n-2$. The last inequality holds since $c \leq \frac{n+1}{n+2}$. Further, for all $r' \in \{1,...,n-2\}$, $\frac{r'}{(r'+2)(r'+1)} \geq \frac{n-2}{n(n'-1)} \geq n-2n(n+1)$. Hence, since $b < \frac{n-2}{n(n+1)}$ and $n \geq 3$, the agent prefers investment $k=1$ to no investment, and so a priori she would not deviate from $k(l) \geq 1$ to $k=0$. This completes the proof for case (2).

(3) $l \in \{0,...,n+1-\lceil c(n+2)\}$. Let us show that in this case there is a continuation equilibrium $k(l)=0$, $r(l)=l$, $j(l)>l$, so that the agent makes zero investment and the project is never approved. Indeed, the agent's strategy $(k(l)=0, r(l)=l)$ is a best response to the principal never approving the project i.e., $j(l)>l$. To confirm that $j(l)>l$ is a best response to $(k(l)=0, r(l)=l)$ we will ascertain that the principal gets a negative payoff when $k(l)=0, r(l)=l$ and the number of successes is $j=l$. Indeed, in this case the principal's payoff is equal to: $\frac{l+1}{l+2}-c$. This expression is negative iff $l < \frac{1}{1-c}-2$. This inequality holds because $l < n+2-\lceil c(n+2)\rceil$ and $n+2-\lceil c(n+2)\rceil < \frac{1}{1-c}-2$. The latter inequality follows since by assumption $c > \frac{n+3-\sqrt{n+3}}{n+2}$. This completed the characterization of the continuation equilibrium in case (3).

To complete the proof, it remains to show that in equilibrium the principal will choose the information acquisition limit $l^e = n+2-\lceil c(n+2)\rceil$. So let us compare the expected payoffs that the principal gets in the continuation equilibria characterized above for each $l \in \{0,...,n\}$ and show that this payoff is maximal at l^e. First, note that for every $l \in \{0,...,n+1-\lceil c(n+2)\}$ (case 3), the principal's expected payoff is zero.

Further, let $\pi(k,j)$ be the principal's expected payoff when the agent invests k, acquires $n-k$ signals and the principal uses threshold j to approve the project. We have:

$$\pi(k,j) = \sum_{j'=j}^{n-k} Pr(j'|k,n)\Big(\frac{k+j'+1}{n+2}-c\Big) = \sum_{j'=j}^{n-k} \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}\Big(\frac{k+j'+1}{n+2}-c\Big) \tag{61}$$

Note that $Pr(j|k,n) = \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}$ is the probability of j successes in $n-k$ signals given investment k.

Consider case (1) now. Then for each $l \in \{\lceil c(n+2)\rceil - 2,...,n\}$, the principal's expected payoff is the same and is given by $\pi(k^*, j(k^*))$ where $k^* = k(\lceil c(n+2)\rceil-2) = (n+2)-\lceil c(n+2)\rceil$, $j(k^*) = \lceil c(n+2)\rceil - k^* - 1 = 2\lceil c(n+2)\rceil - (n+3)$.

Next, in case (2) i.e., for $l \in \{n+2-\lceil c(n+2),...,\lceil c(n+2)-3\rceil\}$, the principal's expected payoff given l is given by $\pi(k(l), j(k(l)))$ where $k(l)=n-l$ and $j(k(l)) = j(l) = \lceil c(n+2)\rceil - k(l) - 1 = \lceil c(n+2)\rceil + l - n - 1$.

Note that our purported equilibrium signal acquisition limit $l^e = n+2-\lceil c(n+2)\rceil$ is the lowest value in the range in case (2), so $k(l^e)) = \lceil c(n+2)\rceil - 2 > k(l)$ for all other l in case 2. Also, $k(l^e)) > k^*$ where $k^* = n - l'$ is the equilibrium investment level in case (1) and $l' = \lceil c(n+2)\rceil - 2$.

Thus, to confirm that $l^e = n+2-\lceil c(n+2)\rceil$ is the equilibrium acquisition limit it is sufficient to show that the principal's expected payoff $\pi(k, j(k))$ increases in k on $\{(n+2) - \lceil c(n+2) ..., \lceil c(n+2)\rceil - 2\}$, where $j(k) = \lceil c(n+2)\rceil - k - 1$.

To this end, we will show that $\pi(k+1, j(k+1)) - \pi(k, j(k)) > 0$ for all $k \in \{1, ..., \lceil c(n+2)\rceil - 3\}$. Indeed, we have:

$$\pi(k+1, j(k+1)) - \pi(k, j(k)) = \sum_{j=j(k)-1}^{n-k-1} Pr(j|k+1,n)\Big(\frac{k+j+1}{n+2} - c\Big)$$
$$- \sum_{j=j(k)}^{n-k} Pr(j|k,n)\Big(\frac{k+j+1}{n+2} - c\Big) = \Big(\frac{k+j(k)+1}{n+2} - c\Big)\sum_{j=j(k)}^{n-k}(Pr(j-1|k+1,n) - Pr(j|k,n))$$
$$+ \frac{1}{n+2}\sum_{j'=j(k)+1}^{n}\sum_{j=j'}^{n-k}(Pr(j-1|k+1,n) - Pr(j|k,n)) =$$
$$\Big(\frac{k+j(k)+1}{n+2} - c\Big)\left(\frac{(k+j(k))!(n-k)!}{(n+1)!(j(k)-1)!} - \frac{(k+j(k))!(n-k-1)!}{(n+1)!(j(k)-2)!}\right)$$
$$+ \frac{1}{n+2}\sum_{j'=j(k)+1}^{n-k}\left(\frac{(k+j')!(n-k)!}{(n+1)!(j'-1)!} - \frac{(k+j')!(n-k-1)!}{(n+1)!(j'-2)!}\right) =$$
$$\Big(\frac{k+j(k)+1}{n+2} - c\Big)\frac{(k+j(k))!(n-k-1)!}{(n+1)!(j(k)-1)!}(n-k-j(k)+1)$$
$$+ \frac{1}{n+2}\sum_{j'=j(k)+1}^{n-k}\frac{(k+j')!(n-k-1)!}{(n+1)!(j'-1)!}(n-k-j'+1) > 0 \tag{62}$$

where the first equality holds by (61), the second equality holds by rearrangement, the third equality holds by (11) in Lemma (5), the forth equality holds by rearrangement, and the inequality holds because all terms in the summations are positive. *Q.E.D.*

Online Appendix

Investment and Information Acquisition

Dimitri Migrow and Sergei Severinov

Generalizing the Payoff Structure

In this section we confirm the robustness of our results by considering more general preferences. Suppose that the agent cares both about the principal's decision to adopt the project, and the net profit from the project, $\theta - c$. Specifically, if the project is adopted by the principal the agent's payoff is

$$(1-\alpha) + \alpha(\theta - c),$$

where $\alpha \in [0,1]$ measures the weight that the agent assigns to the net profit from the project. Thus, α parameterizes the degree of preference alignment between the players. We obtain the following result.

Proposition 6. *For each* $c \in \left[\frac{n+4}{2(n+2)}, \frac{n}{n+2}\right]$ *there exists a non-empty interval* $(0, \alpha'(c)]$*, such that:*

(i) For all $\alpha \in (0, \alpha'(c)]$ *the equilibrium allocation is strictly interior and the investment decreases in the project cost;*

(ii) The upper bound $\alpha'(c)$ *increases in the project cost, so that for a higher cost, there is a larger range of preferences where the result (i) holds;*

(iii) The equilibrium investment weakly increases in α*, as long as* $\alpha'(c) \leq \frac{3}{4}$.

Theorem 6 provides a robustness check of the results in Theorem 1. Particularly, when the alignment between the agent's and the principal's preferences is limited, in the sense that α is sufficiently small, the equilibrium has the same qualitative features as in the baseline model: the agent splits the resources between productive investment and information acquisition, and an increase in project cost leads to a lower productive investment and more information acquisition.

An increase in preference alignment embodied in higher α raises the agent's willingness to make productive investment. This is natural since the principal prefers that all resources are invested. So, a higher α is associated with smaller inefficiency in resource allocation.

Finally, the range of α on which the comparative statics of Theorem 1 holds increases in the project cost. So, when the project cost is sufficiently high, the investment decreases in the project cost even for large α's i.e., more aligned players' preferences.

Proof of the Proposition 6: Recall that in equilibrium the principal approves the project if $j \geq j^* = \lceil c(n+2) \rceil - (k+1)$, where k is the principal's belief about the agent strategy.

Suppose that the agent deviates to investment $k-d$, $d \in \{-(n-k), .., k\}$, while the principal believes that investment level is k. Then, the agent's payoff is

$$Pr(j \geq j^*|k,d)\Big(\alpha(\mathbb{E}[\theta|k,d]-c)+(1-\alpha)\Big),$$

which can be rewritten as:

$$D(k,d,c,\alpha) \equiv \sum_{j=j^*(k)}^{n-(k-d)} \frac{(1+(k-d))(j+(k-d))!(n-(k-d))!}{j!(n+1)!}\left(\alpha\left(\frac{(k-d)+j+1}{n+2}\right)+(1-\alpha)\right) =$$
$$\frac{2-\alpha+k-d+\alpha c(2+k-d)}{2+k-d} -$$
$$\frac{1}{2+k-d}\Bigg(\frac{(n-k+d)!(\lceil c(n+2)\rceil-d-1)!(\alpha(k-d+1)\lceil c(n+2)\rceil}{(n+2)!(\lceil c(n+2)\rceil-(k+2))!}+$$
$$\frac{\alpha(c(n+2)(d-k-2)-(k-d)(n+d)+d-2(k+n+2))+(n+2)(k-d+2))}{(n+2)!(\lceil c(n+2)\rceil-(k+2))!}\Bigg).$$

The equilibrium requires that at the investment level k^*, there is no deviation incentive for the agent to any feasible $k-d$, $d \neq 0$. In other words, the equilibrium condition requires that the function $D(k,d,c,\alpha)$ is maximized at $d=0$. Formally, the following has to be satisfied in an equilibrium:

$$\frac{\partial D(k,d,c,\alpha)}{\partial d}\Big|_{d=0} = 0 \iff D_3(k,c,\alpha) \equiv -\alpha + \frac{(n-k)!(\lceil c(n+2)\rceil-1)!}{(n+2)!(\lceil c(n+2)\rceil-(k+2))!}\times$$

$$\Big[\Big(-\alpha\lceil c(n+2)\rceil(k+1)(k+2)+(2+k)(\alpha(1+c)-1)(2+k)(2+n)\Big)(\psi(n-k+1)-\psi(\lceil c(n+2)\rceil))+$$
$$\alpha\lceil c(n+2)\rceil+(2+k)(\alpha(k+1))\Big] = 0.$$

As we know from the proof of the baseline model, $D(k^*, c, \alpha=0) > 0$, with $k^* = n+2-\lceil c(n+2)\rceil$ (see the proof of Proposition 1).[15] Since for $\alpha = 0$, $D_3(k^*-1, c, \alpha=0) = 0$ which follows directly from the fact that $\psi(n-k+1)-\psi(\lceil c(n+2)\rceil) = 0$ when assuming $k = k^*-1$, it has to be the case that $D_3(k < k^*-1, c, \alpha=0) < 0$ and $D_3(k > k^*, c, \alpha=0) > 0$.

Let us consider k in the domain $[1, n-1]$. Note that as long as there exists $\alpha > 0$ and $k \in [1, .., n-1]$ that solve $D_3(k,c,\alpha) = 0$, we have $\frac{\partial D_3(k,c,\alpha)}{\partial \alpha} < 0$. This is because

$$\frac{\partial D(k,c,\alpha)}{\partial \alpha}\Big|_{d=0} = -\frac{\frac{\Gamma(-k+n+1)((k+1)\lceil c(n+2)\rceil+(-c-1)(k'+2)(n+2))\Gamma(\lceil c(n+2)\rceil)}{\Gamma(n+3)\Gamma(-k+\lceil c(n+2)\rceil-1)}+c(k+2)+1}{k+2} < 0$$

for any feasible k.

This means that as α increases, $D_3(k,c,\alpha)$ decreases, and therefore a larger investment k

[15] Recall that due to the discrete support of $D_3(k,c,\alpha=0)$ in the baseline model, for high enough costs the equilibrium investment is such that $D_3(k^*,c,\alpha=0) \geq 0$.

is required to satisfy the agent's incentive constraint.

To see that there exists a non-empty interval $(0, \alpha(c)]$ that solves $D_3(k, c, \alpha)$ for any $\alpha \in (0, \alpha(c)]$, note that for $\alpha = 0$, $\frac{\partial D_3(k,c,\alpha=0)}{\partial k} > 0$ at $k = k^* - 1$, and for all k in a neighborhood $[k^* - 1 - \delta(c), k^* - 1 + \delta(c)]$, with $\delta(c) > 0$. Since increase in α decreases $D_3(\cdot)$ provided that there exists k, α solving $D_3(k', c, \alpha) = 0$, it must be that there exists a non-empty interval $(0, \alpha(c)]$ where each α in this interval solves $D_3(k', c, \alpha) = 0$.

Next, we show that $D_3(k, c, \alpha)$ increases in c for all α. This means that a lower investment solves the equality. We know that $D_3(k, c, 0)$ increases in c. To see that the derivative is positive, consider again $D_3(k, c, \alpha)$, and note the following. First, $\psi(n-k+1) - \psi(\lceil c(n+2) \rceil)$ decreases in c. The term in front of this difference is negative for $\alpha \leq \frac{3(n+2)}{6(n+1)-2n}$, where $\frac{3(n+2)}{6(n+1)-2n} \geq \frac{3}{4}$. Further, the expression

$$\frac{(n-k)!(\lceil c(n+2) \rceil - 1)!}{(n+2)!(\lceil c(n+2) \rceil - (k+2))!} \alpha \lceil c(n+2) \rceil = \frac{a(n-k)!(\lceil c(n+2) \rceil)!}{(n+2)!(\lceil c(n+2) \rceil - (k+2))!}$$

increases in c for $\alpha > 0$. Therefore, $D_3(k, c, \alpha)$ decreases in c. But then, as long as there exists α that satisfies $D_3(k, c, \alpha) = 0$, the trade-off from the baseline model holds.

Finally, since $D_3(k, c, \alpha)$ increases in c, the interval of α supporting $D_3(k, c, \alpha) = 0$ increases with higher costs. Consider $\alpha'_{max}(c)$. If the project cost increases (and so, the function $D_3(k, c, \alpha)$ increases), then there exists an additional interval for α where the model's results hold. *Q.E.D.*