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The value of information and optimal organization

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The article addresses the issue of optimal organization of production. I compare three organizational forms: centralization (one agent produces different inputs), decentralization (each of two agents produces a different input and contracts directly with the principal), and delegation (two agents produce different inputs, the principal contracts with one of them only). The optimal organizational form depends on the degree of complementarity/substitutability between the inputs in the final use. The degree of complementarity/substitutability also determines whether delegation is payoff equivalent to the two-agent mechanism from the point of view of the principal. In the context of delegation, I consider which of the two agents should serve as the primary contractor. I also address the issue of collusion between the agents in a decentralized organization and characterize the conditions under which a stake of collusion exists.

1. Introduction

■ One of the central issues in the theory of organizations is how information should be distributed, exchanged, and processed within an organization. Answering this question is important for the design of optimal organizational structures. The relevant literature has explored two different approaches in addressing this issue. The first approach focuses on the cost of information processing, whereas the second approach involves studying incentive problems generated by the asymmetry of information between different parties in an organization.

This article contributes to the second strand of literature. It studies an environment where the principal has to implement a project that requires allocating several tasks to subordinates (or, alternatively, procuring several inputs from providers) who have private information regarding the costs of performing these tasks (producing the inputs). The principal has to determine which organizational structure is optimal and design the contracts with subordinates/providers in an optimal way. I adopt the premise that organizational decisions are more durable than production circumstances, so the choice of an organizational structure has to be made before production costs are realized. A number of questions naturally arise in this context. Should several tasks

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(production of different inputs) be centralized in the hands of a single agent (supplier), or should those tasks (production of inputs) be allocated across a number of them? Should the agents be organized in a hierarchy or not, and should the amount of communication between them be restricted? For example, a city council can hire a single contractor for a municipal project, split the work between several firms, or allow the primary contractor to subcontract some work to others. A firm may train its employees as specialists in certain types of tasks, so that several employees typically work on a project. Alternatively, employees may be trained as generalists who can perform tasks of different types and handle all the work on some projects. Similar issues arise in a variety of other contexts, including procurement, outsourcing, and regulation.[1]

To address these issues, I examine three organizational forms in the context of a production process requiring two inputs. In a centralized single-agent organization, one agent supplies both inputs. In a decentralized two-agent organization, each of the two agents supplies a different input. Finally, under delegation, two agents supply different inputs, but the principal contracts with one of them and delegates to her the task of contracting with the second agent. The crucial difference between these organizational forms lies in their informational structure. In the single-agent organization, the agent has private information about production costs of both inputs, in the two-agent organization each agent knows only the cost of one input, whereas under delegation the primary contractor serves as an informational intermediary passing the subcontractor's cost information to the principal. Consequently, the relative profitability of these mechanisms depends on the interaction between these two pieces of information.

Intuitively, the value of information to the agent(s) might be either subadditive or superadditive. In the subadditive case, the value of two pieces of information together, as in the single-agent and delegated mechanisms, is lower than the sum of the values of each piece of information used independently, as in the two-agent mechanism. In the superadditive case, the ordering goes in the opposite way. Put simply, the main issue is whether from an agent's point of view the knowledge of another piece of information increases the value of the first piece of information or decreases it.[2] Because the principal's interests are the opposite of the agent(s)'s interests, the principal prefers informational centralization if the value of information is subadditive for the agents. Conversely, the principal prefers informational decentralization if the value of information is superadditive.

The main insight of this article is that the degree of complementarity or substitutability between the inputs[3] determines whether the value of information is sub- or superadditive. Precisely, under complementarity or small degree of substitutability the value of information is subadditive, provided that the two inputs are not too asymmetric in the final use, and it is superadditive when the degree of substitutability is sufficiently large.

To understand why this is so, consider the value of information in a single-agent mechanism. When the cost of an input is low, the agent earns a rent on this information. The value of this rent is equal to the surplus obtained by misrepresenting this cost as high, and is therefore proportional to the quantity of this input delivered under high cost.

Now consider the effect of misrepresenting a low cost of one input on the value of information about the second input. First, incentive compatibility of the mechanism requires the quantity of an input to decrease in the agent's marginal cost of production. Second, in an efficient ordering, under complementarity (substitutability), the optimal quantity of the second input is increasing

[1] Particularly, while developing a new defense system, the Department of Defense has to decide whether to procure all its components from the same manufacturer or from different ones. The government may allow the existence of a multiproduct monopoly, or break it up into several firms, as in the AT&T case. In more recent examples of deregulation in the electric power industry, the regulators were called to determine whether a public utility producing the bulk of power could also maintain control over the transmission grid or the latter should be controlled by a separate entity.

[2] In the economic literature one can find examples of situations where more information either hurts or benefits the informed party. For example, in the Stackelberg oligopoly game, information about a competitor's action, that is, the competitor's quantity choice, hurts a firm.

[3] These notions are defined below based on the sign of the cross-partial derivative of the principal's benefit function.

(decreasing) in the quantity of the first input. So, under complementarity, misrepresenting the cost of one input upward causes the quantity of the second input to go down, and therefore reduces the informational rent on the second piece of information. Under substitutability, such misrepresentation has the opposite effect because the optimal quantity of an input is increasing in the cost of the second input.

Thus, the reported cost of one input affects the value of information about the cost of the other input. We will refer to this as an "internalization factor," because a single agent internalizes this effect on her total payoff. In contrast, in the two-agent mechanism, each agent exploits the value of her information independently taking the other agent's strategy as given, and this effect is not internalized. Therefore, under complementarity (substitutability), the internalization factor tends to make the value of information subadditive (superadditive).

The other factor affecting the relative performance of the single-agent and two-agent mechanisms is the difference in the structure of incentive constraints. In contrast to the two-agent mechanism, a single agent can manipulate both pieces of information, that is, she can misrepresent production costs of both goods at the same time. So, a larger set of incentive constraints has to be satisfied in the single-agent mechanism. We refer to this as an "extra deviation" factor. This factor makes each piece of information more valuable when the second piece is also known. Hence, it tends to make information superadditive.

To summarize, whether the value of information is sub- or superadditive, and hence which organizational structure is optimal, depends on the relative strength of the internalization and the extra deviation factors. The single-agent mechanism typically dominates the two-agent one under complementarity, because the internalization factor favoring the single-agent mechanism is especially potent in this case. The principal is also able to leverage the effect of the internalization factor and design a mechanism in which the value of information is subadditive under separability and even under a small degree of substitutability. In the latter case, the mechanism involves additional efficiency losses, as the quantity of one input is set to increase in the quantity of the other input—the opposite of the efficient ordering. But because the degree of substitutability is low, these efficiency losses are small, and the single-agent mechanism still dominates the two-agent one.

Nevertheless, the extra deviation factor can overturn the ranking of organizational forms *under complementarity* when there is a strong asymmetry between inputs and the change in quantity of one input affects the marginal product of this input to a lesser degree than the marginal product of the other input. In this case, it becomes very attractive for a single agent to make a joint misrepresentation of the combination of low and high costs as high and low, respectively. Proposition 2 provides the condition under which this extra deviation factor makes the two-agent mechanism more profitable for the principal.

Further, when the degree of substitutability is sufficiently large, it becomes too costly in terms of efficiency losses to use a mechanism in which the quantity of one input increases in the quantity of the other input. But when the ordering of quantities is reversed, the value of information in the single-agent mechanism becomes superadditive because of the extra deviation factor: a low-cost producer of both inputs obtains more profits by misrepresenting both input costs as high. This "coordinated" deviation is infeasible in the two-agent mechanism, so the two-agent mechanism is optimal in this case.

Another interesting set of issues arises in the context of delegation. A delegation mechanism cannot be more profitable than the two-agent mechanism, and the two are equivalent if the primary contractor could not exploit her position of an informational intermediary to increase her profits. Thus, the key issue is whether the primary contractor benefits from intermediating the subcontractor's cost information or simply passes it on to the principal. Potentially, she could benefit from this role in two ways. First, she could try to appropriate some of the subcontractor's informational rent. Second, she could manipulate the report regarding the subcontractor's type to increase the rent on her own information.

I consider four delegation structures which differ in the extent of the principal's contractual abilities. Although the exact conditions under which the two-agent and delegation mechanisms are equivalent vary with the contractual framework, the main conclusion remains the same. The primary contractor benefits from her role of an informational intermediary if the quantity of one input has a significant effect on the marginal product of the other input, that is, if the degree of complementarity or substitutability between the inputs is sufficiently large. To understand this result, note that under these conditions the quantity of the input produced by the primary contractor and hence her informational rent are sensitive to the subcontractor's information. Hence, the primary contractor has stronger incentives to manipulate the latter.

In the context of delegation, I also consider the issue of the optimal choice of the primary contractor. To the best of my knowledge, this issue has never been addressed in the literature before. I identify the conditions determining whom of the two agents the principal should employ as the primary contractor. Specifically, I show that the primary contractor should be the agent who produces an input that has a smaller effect on the marginal product of the other input and who is more likely to be a high-cost producer.

The issues of incentives in organizations and optimal organizational structure have been studied by a number of authors.[4] Baron and Besanko (1992), Gilbert and Riordan (1995), Da Rocha and de Frutos (1999), and Jansen (1999) examine the issue of optimal organization under perfect complementarity between the inputs. Baron and Besanko (1992) and Gilbert and Riordan (1995) show that the single-agent mechanism is superior, and the optimal allocation can also be implemented via delegation.[5] In contrast, Da Rocha and de Frutos (1999) demonstrate that the two-agent mechanism becomes superior under perfect complementarity when the supports of the two cost distributions are sufficiently asymmetric.

Dana (1993) focuses on the effect of correlation in the cost structure under separability of the production function in the two inputs. He shows that the two-agent mechanism is optimal when correlation is sufficiently strong, which allows the principal to exploit relative performance evaluation. Jansen (1999) attains a similar conclusion under perfect complementarity and limited liability assumptions. Demski, Sappington, and Spiller (1987) study the effect of cost correlation on a different organizational choice—optimal input supplier switching. "Informational economies of scope" discussed by Dana under separability are similar to the effect of our internalization factor. Yet, in contrast to his approach, this article focuses on technological interdependency between inputs and its effect on the relative strength of internalization and extra deviation factors.

Perfect complementarity and separability are interesting but quite special cases. Gilbert and Riordan (1995) point out that their analysis of the optimal regulatory regime for the electric power and natural gas industries "depends on the fixed proportions production technology. This is perhaps questionable even in the electricity example, because optimizing the transmission grid may reduce the need for the new generation capacity"; that is, the quality of the grid and the volume of electric power appear to be substitutes. On the other hand, a higher quality of the grid means a higher stability of the network and a lower probability of outages. This may allow consumers to use more electricity and rely less on other forms of energy. So, the same two inputs may be complements. Other examples with some degree of complementarity or substitutability include express and regular mail, long-distance and local telephony, internet and telephone communication, defense systems and municipal projects with multiple components. The results of this article can be applied to obtain conclusions regarding the optimal regulatory regime and optimal purchasing and procurement decisions for these goods and services. Our analysis can also be used to explain the structure of the bicycle manufacturing industry, as well as

[4] Armstrong and Sappington (2004) provide a comprehensive survey of the literature.

[5] Iossa (1999) studies the optimal regulatory regime in a two-good economy with one-dimensional uncertainty with one-dimensional uncertainty about the demand for one of the goods that is privately known by the monopolist or one of the duopolists. She reaches a different conclusion that the regulator prefers monopoly (duopoly) when the goods are substitutes (complements). Given the differences in informational assumptions, the model in this article is not directly comparable to hers.

the trends in enterprise software and procurement decisions in the electronics industry. I discuss these examples in greater detail in Section 2.

In a related contribution, Mookherjee and Tsumagari (2004) study a model with a homothetic benefit function of the principal and a continuous type distribution. They show that the single-agent organization dominates under complementarity when the input costs are identically exponentially distributed, whereas the two-agent organization performs better under substitutability. These results are similar to Propositions 1 and 3 in this article. The difference between their paper and this one boils down to two aspects of the model which, in turn, generate two substantive differences in results. First, the assumption of homotheticity of the benefit function implies a stable relationship between the marginal products of the two inputs which guarantees that nonlocal incentive constraints are never binding in Mookherjee and Tsumagari (2004). In contrast, I allow for an arbitrary benefit function. This leads me to show that, when the benefit function is sufficiently asymmetric, the extra deviation factor becomes effective under complementarity via binding horizontal incentive constraints, and as a result the two-agent mechanism becomes optimal (see Proposition 2).

Second, the definitions of substitutes (complements) in Mookherjee and Tsumagari (2004) are based on the properties of the optimal two-agent (single-agent) mechanism and, thus, do not refer directly to the parameters of the model. In contrast, I define complements and substitutes on the basis of the sign of the cross-partial derivative of the principal's benefit function. Then I show that a single-agent mechanism is optimal under a small degree of substitutability (see Proposition 4). However, it would be impossible to classify this case in Mookherjee and Tsumagari (2004), as it satisfies both their definition of substitutability (the optimal quantity of an input in the two-agent mechanism is increasing in the cost of the other input) and their definition of complementarity (the optimal quantity of an input in the single-agent mechanism is decreasing in the cost of the other input).

The comparison of the single-agent and two-agent mechanisms provides additional insights regarding the potential for collusion in organizations. Laffont and Martimort (1997, 1998) have studied this issue in a similar framework under perfect complementarity. (On the issue of collusion, see also Laffont and Martimort, 2000, and Faure-Grimaund et al., 2003.) They have shown that the potential for collusion exists only under additional restrictions on contracts, such as anonymity. Our results allow us to explain why a stake of collusion does not exist without such restrictions: under complementarity the value of information is typically subadditive, and so the principal prefers informational centralization. Thus, the principal would actually benefit if the agents could collude in the two-agent mechanism and coordinate their strategies to maximize their joint profits.[6] More generally, I show that a stake of collusion always exists under substitutability. Under complementarity, it exists if the two-agent mechanism is optimal (e.g., under the conditions of Proposition 2).

Our analysis of delegation in hierarchial mechanisms is related to the work of Melumad, Mookherjee, and Riechelstein (1995). Of the four delegation mechanisms that we consider, two (H_1 and H_D^{ep}) were first studied by these authors, whereas the other two (H_D and H_1^{ep}) have not been considered previously. Melumad, Mookherjee, and Riechelstein (1995) establish that the delegation mechanism H_1, in which the primary contractor reports her cost to the principal before communicating with the subcontractor and only has to break even in the interim, is equivalent to the two-agent mechanism in the case of a continuous distribution of types. Interestingly, we show that such equivalence does not hold when the set of types is finite. Intuitively, this is due to the fact that in the continuous type case, incentive constraints which involve the primary contractor misrepresenting both her own and the subcontractor's costs hold if the incentive constraints involving a misrepresentation of only one of the two costs are satisfied. However, this is not true in the discrete case under a large degree of substitutability or complementarity (for a more

[6] The principal would then offer them an allocation profile that is implemented in the optimal single-agent mechanism, rather than in a less-profitable two-agent mechanism.

detailed explanation, see footnote 12). In particular, under these conditions, in our model the binding incentive constraint involves the primary contractor reporting her own low cost as high and claiming that the subcontractor's cost is low, irrespective of the true level of the latter, whereas the incentive constraints involving only a misrepresentation of the primary contractor's cost are nonbinding.

As for the hierarchy H_D^{ep}, in which the primary contractor accepts the contract offered by the principal only after contracting with the subcontractor and which is equivalent to hierarchy $H_1^{'}$ in Melumad, Mookherjee, and Riechelstein (1995), the added value of the analysis in this article consists of deriving the exact conditions—in particular, a small degree of complementarity—under which H_D^{ep} attains the performance of the two-agent mechanism.

The two new hierarchies introduced in this article, H_D and H_D^{ep}, capture alternative and realistic scenarios of contracting. In H_D, the primary contractor accepts the principal's contract without reporting her cost. She then contracts with the subcontractor and reports both costs to the principal, but does not have an option to withdraw from the contract after learning the subcontractor's cost. In contrast, in H_D^{ep} the primary contractor first reports her cost to the principal, but can withdraw from the contract at a later stage after receiving the subcontractor's cost report.

The analysis of the single-agent mechanism in this article involves solving a screening problem with a two-dimensional type distributed over a discrete domain, and an arbitrary benefit function of the principal. By characterizing the optimal mechanism in this case and identifying the conditions under which the extra deviation factor is effective and hence nonlocal incentive constraints bind, the article contributes to the literature on multidimensional mechanism design (see Matthews and Moore, 1987; McAfee and McMillan, 1988; Armstrong, 1996; Rochét and Choné, 1998; Wilson, 1993). The paper in this literature that is most closely related is Armstrong and Rochét (1999), who provide a complete characterization of the optimal screening mechanism with two-dimensional agent's type under separability between the goods, but with an arbitrary degree of correlation between the parameters of the agent's type. This article complements theirs, as I characterize the optimal two-dimensional screening mechanism for an arbitrary degree of complementarity or substitutability between the goods but with independently distributed type parameters.

On a more technical side, the contribution of this article lies in demonstrating how the homotopy technique can be applied to compare the performance of different organizational forms. Specifically, I connect the sets of the first-order conditions characterizing the optimal mechanisms in different organizational forms homotopically, that is, via a continuous transformation, and use this to compute the difference between the principal's expected payoffs in the two organizations. I believe that this technique can be used more broadly in the analysis of organizational and contractual problems.

The rest of the article is organized as follows. In Section 2, I discuss several examples and applications of the results presented in this article. In Section 3, I present the model, characterize optimal mechanisms, and describe the results for the symmetric case. Section 4 deals with the complementarity case, and Section 5 deals with the substitutability case. Section 6 studies delegation. Section 7 addresses the issue of collusion. The proofs of Propositions 1 and 2 are in the Appendix. The rest of the proofs are in the online supplement available at http://www.severinov.com/organization_AppendixB.pdf.

2. Examples and applications

■ This section discusses how the results of this article, which will be established in the rest of the paper, can be applied to explain the prevailing forms of organization and to provide recommendations regarding the optimal structure and regulation in several industries.

First, consider the regulation of the electric power industry. As discussed in the Introduction, it is conceivable that the two main components there—the electric power itself and the quality/capacity of the transmission grid—could be either substitutes or complements. This is ultimately an empirical question. In particular, the amount of power and the transmission grid are substitutes if a higher quality of the grid reduces losses and hence demand for the electric power. If the degree of substitutability between these inputs is sufficiently large then, according to Proposition 3, the optimal regulatory regime involves disintegration. It is notable that current regulatory policies in several U.S. states (e.g., California) are gradually moving in this direction. Disintegration is also optimal if these two inputs are complements, but there is a large asymmetry between them (see Proposition 2). This situation is also plausible, because the marginal benefit of an extra unit of power is likely to be as sensitive to an increase in the quality of the grid as to an increase in the volume of energy.

As a related point, this article also suggests (see Proposition 3) that it is optimal to use different providers for regular and express mail, as these are substitute services with a significant degree of substitutability between them. Therefore, the legislative restrictions curbing the ability of the U.S. Postal Service to develop its express mail capabilities could be justifiable.

My results can also be used to explain the regularities in the market for enterprise applications software (often referred to as ERP software). These software applications automate different corporate functions, such as sales, finance, customer relations management, manufacturing, human resources, inventory control, supply chain management, and so on. At the early stages of this market, different software applications were offered for each corporate function, and a limited number of vendors competed in each category, such as Peoplesoft in human resources, SAP in finance, Siebel in sales, i2 in supply chain management, and so on. As the ERP software market evolved and consolidated through mergers, two larger and more successful companies, SAP and Oracle, started offering integrated software serving most corporate functions. However, the market acceptance of such integrated software suites was and remains fairly low. (See Symonds, 2003 for a detailed account of the competition in this industry and the limited success of Oracle's strategy of offering an integrated suite of business applications.) Customers are particularly reluctant to use software from a single supplier for more closely related functions, such as sales, finance, and customer relationship management, or manufacturing and supply chain management. Instead, the corporations are more willing to use software from different providers in related areas, despite the fact that this involves certain functional and data duplication (for example, sales software from one provider and finance software from a different provider would both contain information about order flow and would be capable of producing statistical reports regarding it), and then integrate diverse applications with the help of third-party system integrators, such as IBM or Accenture.

Such customer strategy can be easily explained in the framework of this article. Because software applications for related business areas typically have some degree of overlap and duplication, purchasing such applications from a single source is similar to procuring substitutable inputs from a single supplier. As our results show, the pitfall of this strategy is that a single supplier of such software applications would be able to extract more rent from a customer after the initial purchase—in the form of service and consulting fees, and payments for upgrade modifications—than the sum of what a customer would expect to pay for such services and upgrades to two different suppliers. If the degree of substitutability between the products if sufficiently large, then this factor would be more important than possible economies of scope from using an integrated software suite. So it would, indeed, be optimal for customers to purchase ERP software from different suppliers, consistent with Proposition 3 below. On the other hand, if the degree of substitutability is fairly small, then the customers would be better off with an integrated business software suite, as follows from Proposition 4. So far, the practices in this industry suggest the former to be the case.

The results of this article can also be used to explain the regularities in the bicycle manufacturing industry. Similarly to business software, the bicycle production process has historically been modularized. Different components of a bicycle (frame, tires, wheels, brakes, gears and shifters, pedals, and saddle) are produced independently of each other and then assembled either by specialized designers or by frame manufacturers. Established international standards ensure that the components are interchangeable and can be easily mixed and matched in the final bicycle assembly. So coordination between manufacturers of different components is unnecessary, and innovations are introduced in each component separately, as pointed out by Galvin and Morkel (2001). The number of suppliers of each component does not exceed two or three. For instance, there are only two main producers of gear shifters and moving mechanical parts in the world, Shimano and SRAM, and one fringe supplier—Campy. The situation is similar in the production of other components (for more details, see Chang, Saloner, and Shimano, 2006). Furthermore, antitrust litigation between suppliers has led to elimination of bundling.

With few oligopolistic component manufacturers competing on quality rather than price and quality being the main determinant of the consumers' valuations, this environment fits quite well with the model that I study below. Interestingly, the structure of supplier relationships in the bicycle industry is generally consistent with the predictions of this article. A bicycle normally contains components manufactured by several suppliers. Yet, it is typical for bicycle assemblers, such as Trek, Specialized, or Giant, to procure components that have functional complementarities—such as gear shifters and wheel hubs, headsets, and forks—from a single supplier, although they can easily be procured from different suppliers, as no coordination between suppliers is required. Such practices are consistent with our results saying that it would be more cost effective for a bicycle assembler to procure complementary parts from a single supplier (see Proposition 1).

Finally, the results of Section 6 on delegation can be applied to explain the structure and supplier relationships in vertical supply chains. In particular, in recent years, major suppliers of electronic products (OEMs), such as HP, Motorola, and IBM, have to a large extent outsourced manufacturing to subcontractors. One of the issues that these OEMs have faced is whether they should allow subcontractors to negotiate and purchase materials and components used in production (which is equivalent to the delegation mode in our analysis) or whether the OEMs should negotiate and procure the components themselves and then hand them over to contract manufacturers (which is equivalent to a decentralized mechanism in our analysis). HP has always pursued the latter route. Motorola originally delegated component procurement to subcontractors, but it has recently reversed its policy (see Sullivan, 2003 for details). Motorola found that it could increase its profits by engaging in procurement and directly negotiating with component suppliers. Higher profitability of direct negotiations can be explained by referring to our results. If the contract manufacturers' productivity is significantly affected by the characteristics of the components, that is, the degree of substitutability or complementarity between the subcontractors' inputs and the necessary components is large, then, as Propositions 5–7 predict, it would be optimal for OEMs to procure the components directly, rather than to delegate. The reversal of Motorola's policy can be interpreted to indicate that this is, indeed, so.

3. Model and preliminaries

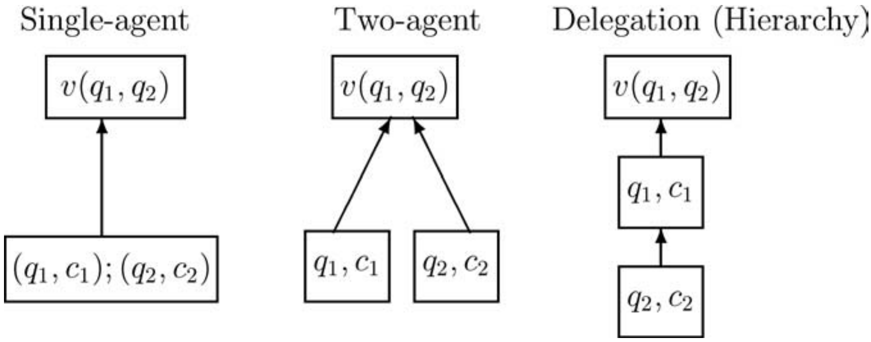
■ A central entity, or principal, needs to procure two different goods or inputs. The principal's benefit is measured by the production/benefit function $v(q_1, q_2)$, where q_i is the quantity of input i, for $i \in \{1, 2\}$. I assume that $v(.,.)$ is increasing in both arguments, twice continuously differentiable, and concave. The cross-partial derivative $v_{12}(.,.)$ has a constant sign over the relevant domain. We will say that the inputs are complements (substitutes) if $v_{12}(.,.) \geq 0$ ($v_{12}(.,.) < 0$). To ensure that the optimal quantities are positive, I impose the Inada boundary condition: $\lim_{q_1 \to 0} v_1(q_1, q_2) = \infty$ for all $q_2 > 0$. This condition is dropped when I consider specific examples.

I will compare the performance of three organizational forms illustrated in Figure 1: centralized organization (one agent produces both inputs), decentralized organization (each input is produced by a different agent), and delegation mechanism where the agents are organized in a hierarchy and the principal contracts only with the supplier of one input, who in turn contracts with the supplier of the second input.[7] In each organizational form, the principal offers a contract(s) to the agent(s), who may either accept or reject the offer. If the contract(s) is (are) accepted,

[7] There is a number of reasons why the principal may want or have to procure all supply of a particular input from one source. The most common of them is the presence of fixed costs. If large fixed costs in the form of R&D, investment

FIGURE 1

THREE ORGANIZATIONAL FORMS



the agent(s) produces and delivers the goods/inputs to the principal and gets paid according to the contract(s). Additional contracting stages in the delegation mechanism are described in Section 6.

The principal maximizes her expected benefit net of the expected payments for the inputs. The agents(s) are risk neutral and decide whether to accept a contract after privately learning their production cost(s). An agent's reservation utility level is normalized to zero. An agent cannot produce the good which she is not assigned to. The marginal costs of production are constant and are independently distributed across goods and across agents. Specifically, it is common knowledge that the marginal cost of good i is low (c_L) with probability p_i, and is high (c_H) with the complementary probability, where $c_H > c_L > 0$. Let $\Delta = c_H - c_L$. Because the benefit/production function $v(., .)$ can be arbitrarily asymmetric, the assumption that the distributions of input costs have a "common support" is equivalent to a less-restrictive "common ratio" assumption $\frac{c_L^1}{c_H^1} = \frac{c_L^2}{c_H^2}$ from which "common supports" can be obtained by simple renormalization of units. Independence of distributions is assumed in order to abstract from factors on the cost side.

Let us now describe the contracts offered by the principal in the single-agent and the two-agent mechanisms. By the Revelation Principle (see e.g., Baron, 1989), we can restrict attention to incentive-compatible direct mechanisms in which every agent reports her cost truthfully. Recall that a direct mechanism is a mapping from the set of possible cost types $\{c_L, c_H\} \times \{c_L, c_H\}$ (or states of the world) into the set of quantities and transfers: $R_+^2 \times R^2$ (in two-agent mechanism) or $R_+^2 \times R$ (in a single-agent mechanism). The four possible states of the world are denoted by LL, LH, HL, and HH. In this notation, the first (second) letter indicates the marginal cost of the first (second) good.

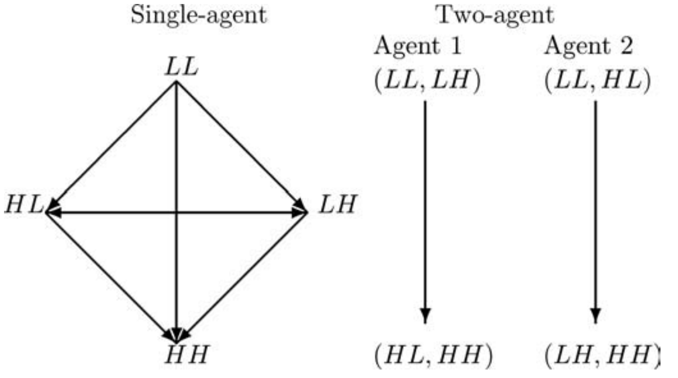
Let $\mathbf{q}^i = (q_{LL}^i, q_{LH}^i, q_{HL}^i, q_{HH}^i)$ denote the vector of quantities of good $i \in \{1, 2\}$ assigned in the two-agent mechanism. By convention, the first letter in the subscript refers to the marginal cost of good i. For example, in the state LH, the mechanism assigns quantities q_{LH}^1 and q_{HL}^2. Let t_{KJ}^i denote the transfer to the agent producing good i, in the case when she announces cost c_K and

in equipment, infrastructure, and training, and so forth, have to be sunk by each producer of the good before she learns her production costs, then having more than one supplier could be prohibitively expensive. Alternatively, the principal's commitment to purchase all supply of an input from a particular agent may be required to alleviate a potential hold-up problem and induce this agent to make necessary investments, or to perform R&D.

Consider, for example, the development of a new defense system. In the initial stage of procurement, the government normally considers bids from a number of suppliers. However, only one supplier of each major part is ultimately chosen. Moreover, the final price is usually determined after the contracts have already been awarded. According to Rogerson (1989), "economies of scale together with very small production runs render it economically infeasible to have two or more firms build fully functioning production lines. . . . The prices for all production runs may be left to be determined by future negotiations. Transaction costs together with constantly evolving technological requirements are thought to render long-term contracts infeasible."

FIGURE 2

INCENTIVE CONSTRAINTS IN THE SINGLE-AGENT AND TWO-AGENT MECHANISMS



the other agent announces cost $c_J (K, J \in \{L, H\})$. The two-agent mechanism has to satisfy the following interim incentive and individual rationality constraints for each i and $j \in \{1, 2\}$, $i \neq j$:

$$\begin{aligned} IC^i(L) : &\left(t^i_{LL} - c_L q^i_{LL}\right)p_j + \left(t_{LH} - c_L q^i_{LH}\right)(1 - p_j) \geq \left(t^i_{HL} - c_L q^i_{HL}\right)p_j \\ &+\left(t_{HH} - c_L q^i_{HH}\right)(1 - p_j) \\ IC^i(H) : &\left(t^i_{HL} - c_H q^i_{HL}\right)p_j + \left(t_{HH} - c_H q^i_{HH}\right)(1 - p_j) \geq \left(t^i_{LL} - c_H q^i_{LL}\right)p_j \\ &+\left(t_{LH} - c_H q^i_{LH}\right)(1 - p_j) \\ IR^i(L) : &\left(t^i_{LL} - c_L q^i_{LL}\right)p_j + \left(t_{LH} - c_L q^i_{LH}\right)(1 - p_j) \geq 0 \\ IR^i(H) : &\left(t^i_{HL} - c_H q^i_{HL}\right)p_j + \left(t_{HH} - c_H q^i_{HH}\right)(1 - p_j) \geq 0. \end{aligned}$$

Next consider a single-agent mechanism. Let $\mathbf{g}^i = (g^i_{LL}, g^i_{LH}, g^i_{HL}, g^i_{HH})$ denote the vector of quantities of good i assigned in this mechanism, and T_{KJ} denote the transfer to the agent who announces costs (c_K, c_J), where $K, J \in \{H, L\}$. The single-agent mechanism has to satisfy the following incentive and individual rationality constraints for all $K, J, U, V \in \{L, H\}$:

$$IC(KJ - UV) : \quad T_{KJ} - c_K g^1_{KJ} - c_J g^2_{JK} \geq T_{UV} - c_K g^1_{UV} - c_J g^2_{VU}$$

$$IR(KJ) : \quad T_{KJ} - c_K g^1_{KJ} - c_J g^2_{JK} \geq 0.$$

The structures of incentive constraints in the two-agent and single-agent mechanisms are depicted in Figure 2, ignoring upward incentive constraints, which, as I show, never bind in an optimal mechenism. The downward incentive constraint $IC(LL - HH)$, as well as the horizontal incentive constraints $IC(LH - HL)$ and $IC(HL - LH)$ in the single-agent mechanism have no counterparts in the two-agent mechanism, because agents choose their reports independently in the latter one. When any one of these constraints is binding, it reduces the profitability of the single-agent mechanism, that is, the extra deviation factor is effective. On the other hand, the constraints $IC(LL - HL)$, $IC(LL - LH)$, and $IC(LL - HH)$ in the single-agent mechanism are mutually exclusive, and so the principal can ensure that all these three constraints hold by paying the agent a single informational rent in state LL. This is a manifestation of the internalization factorThe main results of this article, which will be established in the following sections, show that the optimal organizational form depends on the degree of complementarity and substitutability between the inputs measured by the ratios $|\frac{v_{12}(g_1, g_2)}{v_{11}(g_1, g_2)}|$ and $|\frac{v_{12}(g_1, g_2)}{v_{22}(g_1, g_2)}|$. These ratios provide an appropriate measure of complementarity and substitutability because they determine the relative rate at which the quantities of the two inputs change in response to a change in the cost of one of the inputs. Specifically, if $|\frac{v_{12}(g_1, g_2)}{v_{ii}(g_1, g_2)}|$ is large, then a change in the cost of jth input, $j \neq i$, causes the optimal quantity of input i to change by a relatively large amount compared to the change in the quantity of input j.

□ **Optimal two-agent mechanism.** As a first step in the analysis, I characterize, the optimal two-agent mechanism. The optimal single-agent mechanism turns out to be quite sensitive to the degree of complementarity and substitutability. Therefore, this mechanism is characterized separately under complementarity (see the proof of Proposition 2) and under substitutability (see the proof of Proposition 3).

In the rest of this subsection, I consider the optimal two-agent mechanism. Essentially, it consists of two submechanisms, one for each agent. In each of them, the individual rationality constraint of the high-cost type and the incentive constraint of the low-cost type are binding. Substitutability and complementarity cause the quantity assigned to one of the agents to depend on the cost type of the other agent, but does not affect the set of binding constraints.

Lemma 1. The optimal two-agent mechanism is unique. The optimal quantities are determined by the following first-order conditions:

$$v_1\left(q^1_{LL}, q^2_{LL}\right) = v_2\left(q^1_{LL}, q^2_{LL}\right) = v_1\left(q^1_{LH}, q^2_{HL}\right) = v_2\left(q^1_{HL}, q^2_{LH}\right) = c_L \tag{1}$$

$$v_1\left(q^1_{HL}, q^2_{LH}\right) = c_H + \Delta\frac{p_1}{1-p_1} \tag{2}$$

$$v_2\left(q^1_{LH}, q^2_{HL}\right) = c_H + \Delta\frac{p_2}{1-p_2} \tag{3}$$

$$v_1\left(q^1_{HH}, q^2_{HH}\right) = c_H + \Delta\frac{p_1}{1-p_1} \tag{4}$$

$$v_2\left(q^1_{HH}, q^2_{HH}\right) = c_H + \Delta\frac{p_2}{1-p_2}. \tag{5}$$

The optimal quantity of an input is:

(i) decreasing in its cost, that is, $q^i_{LL} > q^i_{HL}, q^i_{LH} > q^i_{HH}$;
(ii) decreasing in the cost of the other input, that is, $q^i_{LL} > q^i_{LH}$ *and* $q^i_{HL} > q^i_{HH}$, under complementarity;
(iii) increasing in the cost of the other input, that is, $q^i_{LL} < q^i_{LH}$ *and* $q^i_{HL} < q^i_{HH}$, under substitutability;
The transfers are given by $t^i_{HK} = c_H q^i_{HK}$, $t^i_{LK} = c_L q^i_{LK} + \Delta q^i_{HK}$ for $K \in \{L, H\}$.

Thus, to reduce the agents' informational rents, the principal sets quantity allocations q^i_{HL} and q^i_{HH} in the two-agent mechanism below the first-best. The quantities q^i_{LL} are set at the first-best level (no distortion "at the top"), whereas q^i_{LH} is set above (below) the first-best level when the inputs are substitutes (complements).

4. Complementarity

■ In this section, I compare the profitability of the single-agent and the two-agent mechanisms under complementarity. The outcome of this comparison depends on the degree of complementarity. The following proposition shows that the single-agent mechanism dominates when the degree of complementarity is not too large.

Proposition 1. Suppose that the inputs are complementary, that is, $v_{12}(.,.) \geq 0$. Then the single-agent mechanism is more profitable for the principal than the two-agent mechanism if the degree of complementarity between the inputs is not too large, that is, $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| \leq 1$ *for all* $i \in \{1, 2\}, (q_1, q_2) \in \mathbb{R}^2_+$.

When the degree of complementarity is less than 1, the value of information is subadditive and the principal can implement the quantity profile from the optimal two-agent mechanism via a single-agent mechanism with lower expected payments. Specifically, in states *HH*, *LH*, and *HL*, the payments in the single-agent mechanism can be set equal to the sum of payments in the two-agent mechanism, whereas in state LL, a lower payment can be made in the single-agent mechanism due to the internalization factor.

To see the latter, note that in state LL, the total informational rent paid by the principal in the two-agent mechanism is equal to $\Delta(q^1_{HL} + q^2_{HL})$, because each agent can independently misrepresent her cost as high. If the same allocation profile is assigned in the single-agent mechanism, then the agent can deviate by misrepresenting only one input cost, or the costs of both inputs. The latter deviation is least attractive, because under complementarity the optimal quantity of one input decreases in the cost of the other input and, in particular, $q^i_{HL} > q^i_{HH}$ by Lemma 1. If the agent misrepresents only the cost of the ith input, for $i \in \{1, 2\}$, then she earns a rent equal to Δq^i_{HL} on her information regarding the cost of this input, but her rent on the information regarding the cost of the input j, $j \neq i$, will be at most Δq^j_{HH}. So, in state LL in the single-agent mechanism, the principal needs to pay informational rent equal to $\Delta \max \{q^1_{HL} + q^2_{HH}, q^1_{HH} + q^2_{HL}\}$, which is less than the informational rent $\Delta(q^1_{HL} + q^2_{HL})$ paid in the two-agent mechanism. Thus, the value of information is subadditive. Finally, the restriction that the degree of complementarity should not exceed 1 ensures that the horizontal incentive constraints $IC(LH - HL)$ and $IC(HL - LH)$ remain nonbinding in the single-agent mechanism.

When the degree of complementarity is sufficiently large and there is a strong asymmetry between the inputs, then an increase in the quantity of one input affects the marginal product of this input to a lesser degree than the marginal product of the other input. Consequently, one of the horizontal incentive constraints becomes binding, and the result of Proposition 1 may be reversed due to this extra deviation factor. A sufficient condition for this is given in the following proposition. It is stated in terms of the inverses of the degrees of complementarity.

Proposition 2. The two-agent mechanism is more profitable than the single-agent mechanism under complementarity if for some i and $j \in \{1, 2\}$, $i \neq j$, the following condition holds for all $(q_1, q_2) \in \mathbb{R}^2_+$:

$$-\frac{v_{jj}(q_1, q_2)}{v_{12}(q_1, q_2)}(1 - p_j)^2 p_i - \frac{v_{ii}(q_1, q_2)}{v_{12}(q_1, q_2)}(p_i(1 - p_j) + p_j) < ((1 - p_j)(2p_i + p_j(1 - p_i)). \quad (6)$$

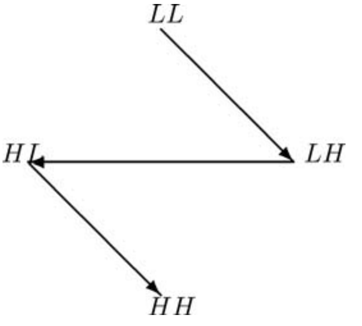
Condition (6) requires the degree of complementarity to be sufficiently large. In particular, combining (6) with the fact that $v(.)$ is concave, we obtain that $|\frac{v_{12}(.)}{v_{ii}(.)}| > \frac{1}{1-p_j}$, that is, the degree of complementarity exceeds $\frac{1}{1-p_j}$. To understand the implications of this condition for the single-agent mechanism, suppose that $i = 2$. Then, as we move from state HH to state LH, that is, as the marginal cost of the first input goes down while the cost of the second input remains high, the optimal quantity of the first input in the single-agent mechanism increases by a smaller increment than the optimal quantity of the second input. As a result, the incentive constraint $IC(HL - LH)$ becomes binding, as it gets more attractive for the agent in state HL to misrepresent both costs and report LH. By doing so, the agent sustains a small loss on the first input, because her true cost of producing this input is high, but she earns a large informational rent on her low cost of the second input. This extra deviation factor makes the two-agent mechanism more profitable, because the nonlocal incentive constraint $IC(HL - LH)$ does not have to hold in the two-agent mechanism.

Proposition 2 is proven by showing that a relaxed single-agent mechanism, in which constraints $IC(HL - HH)$, $IC(HH - LH)$, and $IC(LL - HH)$ are omitted and which therefore is more profitable for the principal than the optimal single-agent mechanism, is dominated by a two-agent mechanism. When condition (6) holds, the set of binding incentive constraints in this relaxed single-agent mechanism consists of $IC(LL - HL)$, $IC(HL - LH)$, and $IC(LH - HH)$ (see Figure 3).[8]

[8] In the online supplement available at http://www. severinov.com/organization_AppendixB.pdf, I show that the relaxed mechanism satisfies the omitted incentive constraints and hence is equivalent to the optimal single-agent mechanism if, in addition to condition (6), we also require that $v_{12} p_i(1 - p_j) + v_{ii}(p_i(1 - p_j) + p_j) \leq 0$.

FIGURE 3

THE SET OF BINDING INCENTIVE CONSTRAINTS UNDER THE CONDITION OF PROPOSITION 2



In the special case of the quadratic benefit function, we can use the proofs of Propositions 1 and 2 to obtain a necessary and sufficient condition for optimality of a particular organizational structure under complementarity. As in Proposition 2, this condition is also stated in terms of the inverses of the degrees of complementarity. It is slightly different from (6) because in the quadratic case, we can directly compute the expressions for the difference in expected payoffs between the single-agent and the two-agent mechanisms rather than just bound them, as in the general case.

Corollary 1. Suppose that $v(q_1, q_2) = A + a(q_1 + q_2) - \frac{b_1}{2}q_1^2 - \frac{b_2}{2}q_2^2 + dq_1q_2$, *where* A, a, b_1, b_2, d are positive constants (so that $v_{12} = d > 0$) satisfying $b_1 b_2 \geq d^2$. Then the two-agent mechanism is optimal if and only if for some i and $j \in \{1, 2\}$, $i \neq j$, we have

$$\frac{b_j}{d}\frac{(1-p_j)^2 p_i}{2} + \frac{b_i}{d}\left(p_j(1-p_i) + \frac{p_i}{2}\right) < (1-p_j)(p_j + p_i(1-p_j)). \qquad (7)$$

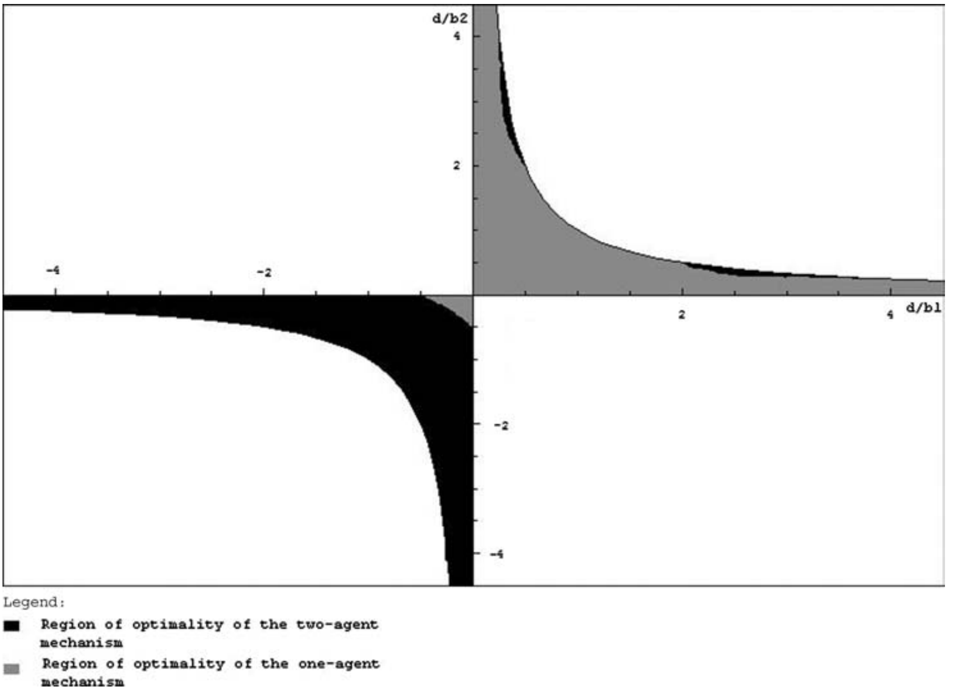
A graphical illustration of Corollary 1 is provided in Figure 4 for the case $p_1 = p_2 = 1/2$. In particular, its northeastern quadrant corresponds to the complementarity region, $d > 0$. The upper boundary of the admissible parameter space in this region is given by $b_1 b_2 = d^2$ (concavity restriction), while the lower boundary of the region of optimality of the two-agent mechanism is given by (7) holding as equality. Furthermore, under the normalization $d = 1$, with $i = 2$ and $j = 1$, the inequality (7) holds and $b_1 b_2 > 1$ when $b_1 = \frac{1+\epsilon_1}{1-p_1} > 0$ and $b_2 = (1 - p_1)(1 - \epsilon_2) > 0$, where both ϵ_1 and ϵ_2 are positive numbers satisfying $\epsilon_1 + 1 < \frac{\epsilon_1}{\epsilon_2} < \frac{2p_1(1-p_2)}{p_2} + 1$.

Da Rocha and de Frutos (1999) establish a result related to our Proposition 2. They show that the two-agent mechanism can outperform the single-agent mechanism under perfect complementarity. These authors emphasize the asymmetry of the supports of the cost distributions (i.e., is $\frac{c_H^1 - c_L^1}{c_H^2 - c_L^2}$ sufficiently larger than 1) as an explanation. Yet, as our analysis indicates, a strong complementarity in their production function must also play a role in their result. Indeed, Proposition 1 implies that, for any value of $\frac{c_H^1 - c_L^1}{c_H^2 - c_L^2}$, the single-agent mechanism remains optimal when the degree of complementarity is sufficiently small.[9] Conversely, performing renormalization, it is easy to show that the result of Da Rocha and de Frutos (1999) also holds when the cost distributions have a common support, and the production/benefit function is given by $\min\{\frac{q_1}{r_1}, \frac{q_2}{r_2}\}$ when $\frac{r_1}{r_2}$ is large enough. This condition is similar to condition (6) in Proposition 2. (It is not identical because of the nondifferentiability of the Leontieff production function at the corner points.)

[9] As pointed out in Section 3, all the results of this article hold if we replace the common support assumption with the common ratio assumption $\frac{c_L^1}{c_H^1} = \frac{c_L^2}{c_H^2}$. In turn, an arbitrary large $\frac{c_H^1 - c_L^1}{c_H^2 - c_L^2}$ is consistent with the common ratio assumption.

FIGURE 4

REGIONS OF OPTIMALITY UNDER QUADRATIC BENEFIT FUNCTION. ASYMMETRIC CASE



5. Substitutability

■ Compared to the complementarity, there are several differences in the nature and strength of the internalization and extra deviation factors under substitutability. The main reason for this is that, under substitutability, the efficiency requires the quantity of one input to increase in the cost of the other input. In particular, it is efficient to set $g^i_{HH} > g^i_{HL}$ for $i \in \{1, 2\}$ in the single-agent mechanism. If the quantity profile in the single-agent mechanism satisfies this ordering, then the extra deviation factor manifests itself in the form of binding incentive constraint $IC(LL - HH)$, that is, the most profitable deviation for the agent with two low costs is to report that both are high.

In the two-agent mechanism, the principal does not need to be concerned about this deviation because the agents could not coordinate their strategies. So, if $IC(LL - HH)$ is binding in the single-agent mechanism, then the value of information is superadditive, and the two-agent mechanism dominates. This is shown in the proof of Proposition 3 in the Appendix.

Still, the potential to exploit the internalization factor can make it optimal for the principal to violate the efficient ordering in the single-agent mechanism and implement a profile of quantities decreasing in the cost of the other input, in particular, by setting $g^i_{HL} > g^i_{HH}$ for $i \in \{1, 2\}$. Then the value of information will be subadditive in the single-agent mechanism, as the principal will pay a lower informational rent than in the two-agent mechanism with the same quantity assignment. This, however, will be achieved at the cost of productive distortions. In contrast, by Lemma 1, the quantity profile in the two-agent mechanism under substitutability is always increasing in the cost of the other input, which is more efficient. In this case, the optimal organizational form is determined by the tradeoff between a lower informational rent in the single-agent mechanism and a higher efficiency of the two-agent mechanism.

We use the homotopy technique to determine which of these two factors dominates. Similarly to the complementarity case, the results depend on the degree of substitutability measured by

$|\frac{v_{12}(g_1,g_2)}{v_{11}(g_1,g_2)}|$ and $|\frac{v_{12}(g_1,g_2)}{v_{22}(g_1,g_2)}|$. Also, the threshold degree of substitutability above which the two-agent mechanism dominates turns out to depend on the relative likelihood of low- and high-cost states of the world.

At first, we will focus on the conditions under which the two-agent mechanism is optimal. Let $\underline{g}_1, \bar{g}_2$ solve $v_1(\underline{g}_1, \bar{g}_2) = c_H + \Delta\frac{p_1}{(1-p_1)(1-p_2)}$ and $v_2(\underline{g}_1, \bar{g}_2) = c_L$. Also, let $\bar{g}_1, \underline{g}_2$ solve $v_1(\bar{g}_1, \underline{g}_2) = c_L$ and $v_2(\underline{g}_1, \bar{g}_2) = c_H + \Delta\frac{p_2}{(1-p_1)(1-p_2)}$. Then we have the following.

Proposition 3. Suppose that the inputs are substitutes and let $r = \min_{(g_1,g_2)\in[\underline{g}_1,\bar{g}_1]\times[\underline{g}_2,\bar{g}_2]} |\frac{v_{12}(g_1,g_2)}{v_{11}(g_1,g_2)}|$. Then the two-agent mechanism is optimal if either $r \geq \frac{p_2}{1-p_2}$ or $\min_{(g_1,g_2)\in[\underline{g}_1,\bar{g}_1]\times[\underline{g}_2,\bar{g}_2]} |\frac{v_{12}(g_1,g_2)}{v_{22}(g_1,g_2)}| \geq \frac{p_1}{p_2}\frac{p_2-r(1-p_2)}{rp_1(2-p_1-p_2+p_1p_2)+(1-p_1)}$.

According to Proposition 3, the two-agent mechanism is optimal whenever the degree of substitutability between the inputs is sufficiently high. In this case, the quantity distortions (compared to the efficient levels) needed to neutralize the extra deviation factor in the single-agent mechanism become too large. The principal then sets $g^i_{HH} \geq g^i_{HL}$ for all $i \in \{1, 2\}$ in the single-agent mechanism, making the constraint $IC(LL - HH)$ binding, and so the two-agent mechanism becomes more profitable. Furthermore, the threshold degrees of substitutability $\frac{p_2}{1-p_2}$ and $\frac{p_1}{p_2}\frac{p_2-r(1-p_2)}{rp_1(2-p_1-p_2+p_1p_2)+(1-p_1)}$ are increasing in p_2 and in both p_1 and p_2, respectively. This is so because the state LL is more likely when both p_1 and p_2 are high, and the internalization factor makes the single-agent mechanism more profitable for the principal precisely in state LL.

The following corollary shows that the degree of substitutability required for the two-agent mechanism to be optimal is less than 1.

Corollary. For all $p_1, p_2 < 1$, there exists $k < 1$ s.t. the two-agent mechanism is optimal under substitutability if $\min\{|\frac{v_{12}(g_1,g_2)}{v_{11}(g_1,g_2)}|, |\frac{v_{12}(g_1,g_2)}{v_{22}(g_1,g_2)}|\} \geq k$ for all $(g_1, g_2) \in [\underline{g}_1, \bar{g}_1] \times [\underline{g}_2, \bar{g}_2]$.

Corollary 2 is immediately applicable to the case of perfect substitutes, that is, when $v(q_1, q_2) = u(q_1 + q_2)$. In this case, $\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)} \equiv 1$ for $i \in \{1, 2\}$, so the two-agent mechanism is optimal.

Next, suppose that the degree of substitutability is low. Then, in the single-agent mechanism, the principal exploits the internalization factor by making the quantity of one input decrease in the cost of the other input—much like under complementarity. However, in the two-agent mechanism we have the opposite ordering: the optimal quantity of one input is increasing in the cost of the other input. Because the optimal quantities are ordered differently in the single-agent and the two-agent mechanisms, a simple method of proof based on the comparison of informational rents is not applicable. Instead, to compare the principal's expected profits in the optimal single-agent and two-agent mechanisms, we use the homotopy technique developed in the Appendix. The following proposition describes the result of this comparison.

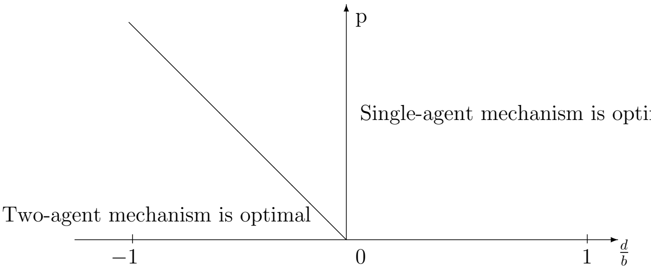
Proposition 4. Suppose that the inputs are substitutes and that there exist $\underline{K}$ and $\bar{K}$, $0 < \underline{K} \leq \bar{K} < \infty$, s.t. $\underline{K} < v_{11}(q_1, q_2)/v_{22}(q_1, q_2) < \bar{K}$ for all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$. Then for any $(p_1, p_2) \in (0, 1)^2$ there exist ω_1 and ω_2, with ω_i increasing in p_j, $i \neq j$, such that the single-agent mechanism is optimal if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| < \omega_i$ for all $i \in \{1, 2\}$ and $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$.

Proposition 4 holds because the effect of the internalization factor outweighs the efficiency losses from distorting the quantity profile in the single-agent mechanism when the degree of substitutability is low, and when both p_1 and p_2 are sufficiently high so that state LL occurs with a high likelihood. Low substitutability implies that the efficiency losses from exploiting the internalization factor and making the quantity profile decrease in the cost of the other input in the single-agent mechanism (in particular, setting $g^i_{HL} > g^i_{HH}$ for $i \in \{1, 2\}$) is not too large, whereas the high likelihood of state LL makes the effect of the internalization factor sufficiently large in expected terms.

In the symmetric case, that is, when $v(q_1, q_2) = v(q_2, q_1)$ for all $(q_1, q_2) \in \mathbf{R}^2_+$ and $p_1 = p_2 = p$, we can obtain somewhat tighter bounds on the regions of optimality of the single-agent and two-agent mechanisms under substitutability. Revisiting the proofs of Propositions 3 and

FIGURE 5

REGIONS OF OPTIMALITY UNDER QUADRATIC BENEFIT FUNCTION. SYMMETRIC CASE, WITH $|\frac{d}{b}| < 1$



4 under symmetry,[10] we obtain that the single-agent mechanism remains optimal if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| \leq \min\{\frac{p}{4}, \frac{p^2}{4(1-p)+p^2}\}$, whereas the two-agent mechanism is optimal if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| \geq \frac{p}{2(1-p)+p^2}$ for all $(q_1, q_2) \in \mathbf{R}^2_+$ and $i \in \{1, 2\}$.

Finally, in the special case of the quadratic benefit function, we can use the proofs of Propositions 3 and 4 to derive a necessary and sufficient condition for the optimality of each organizational structure under substitutability. It illustrates the results of these propositions by showing exactly how large the degree of substitutability has to be for the two-agent mechanism to dominate.

Corollary 3. Suppose that $v(q_1, q_2) = A + a(q_1 + q_2) - \frac{b_1}{2}q_1^2 - \frac{b_2}{2}q_2^2 + dq_1q_2$, where A, a, b_1, b_2 are positive constants, d is negative (substitutability), and $b_1b_2 \geq d^2$ (concavity). Then the two-agent mechanism is more profitable than the single-agent mechanism if and only if

$$\frac{d^2}{b_1b_2}\frac{(p_1+p_2-p_1p_2)^2}{p_1p_2} - \frac{d}{b_2}\frac{2(1-p_1)}{p_1} - \frac{d}{b_1}\frac{2(1-p_2)}{p_2} \geq 1. \tag{8}$$

Figure 4 provides a graphical illustration of Corollary 3 for $p_1 = p_2 = 1/2$. The lower-left quadrant in this figure corresponds to the substitutability region, where $d < 0$. The lower boundary of the admissible parameter space in this region is given by $b_1b_2 = d^2$ (concavity restriction), while the upper boundary of the region of optimality of the two-agent mechanism is given by (8) holding as equality.

Finally, Figure 5 illustrates the regions of optimality of the two-agent and the single-agent mechanisms with symmetry and quadratic benefit function, that is, when $b_1 = b_2$ and $p_1 = p_2$. Note that in this case condition (8) simplifies to $|\frac{d}{b}| \geq \frac{p}{4(1-p)+p^2}$.

6. Delegation

■ In this section, I consider another form of organization—delegation, with agents organized in a hierarchy. The two agents contribute different inputs, but the principal directly contracts only with the primary contractor and delegates to her the task of contracting with the other agent, the subcontractor (see Figure 1). Delegation is common in the allocation of tasks within an organization, in procurement and in the construction industry. In large corporations, senior managers typically delegate some supervisory functions and authority to middle managers.

Hierarchial delegation with asymmetric information was studied by a number of authors, in particular, Melumad, Mookherjee, and Riechelstein (1995), Baron and Besanko (1992), Gilbert and Riordan (1995), Laffont and Martimort (1998), and Mookherjee and Riechelstein (2001). This literature points out that the advantages of delegation include an economy of communication costs

[10] In this case, the multiplier α in the proofs of these propositions is equal to 1/2, which allows us to simplify the derivations significantly.

achieved by shifting some of the contracting tasks from the principal to one of the subordinates.[11] On the other hand, delegation leads to a loss of control by the principal which may negatively affect the incentives within hierarchies. The last point is made by McAfee and McMillan (1995) in the context of a model where intermediate layers of supervision separate the principal from the agent engaged in production. Riordan and Sappington (1987) show that the principal's decision whether to delegate both stages of the production process to the agent or only one stage depends on whether the costs at the two stages are positively or negatively correlated.

This section has two goals. The first goal is to compare the profitability of the delegation mechanism vis-à-vis the two-agent and the single-agent mechanisms in several contractual environments. The second goal is to examine the issue of the optimal choice of the primary contractor. In our model, agents 1 and 2 can have different cost distributions and different productivities, as reflected in the asymmetry of the production function. It is natural to ask whether these asymmetries imply differential performance by agents 1 and 2 in the role of primary contractor.

To make legitimate comparisons across organizational forms, I make the same assumptions regarding input observability as in the single-agent and the two-agent organizations studied in the previous sections. Specifically, I assume that under delegation, the principal can monitor the quantity of an input supplied by each agent. I will consider four different contractual setups referred to as delegation hierarchies H_1, H_D, H_D^{ep}, and H_D^{ep}. The following sequence of moves characterizes hierarchy H_1 (named so by Melumad, Mookherjee, and Riechelstein 1995):

(i) The principal offers the contract to the primary contractor.
(ii) The primary contractor decides whether to accept or reject the contract. If she rejects, the game ends and all players obtain their reservation payoffs. If the primary contractor accepts the contract, then the game proceeds through the following stages.
(iii) The primary contractor reports her cost type to the principal.
(iv) The primary contractor offers a contract to the subcontractor. If the subcontractor rejects it, then the game ends and all players obtain their reservation payoffs.
(v) If the subcontractor accepts, she reports her cost to the primary contractor, who then reports it to the principal.
(vi) Both contractors produce their inputs, the final output is delivered to the principal, and the transfers take place according to the two contracts.

The hierarchy H_1 is the most profitable for the principal among all delegation hierarchies with the same observability assumptions, because it endows the principal with the broadest possible contracting abilities. In particular, the principal signs a contract with the primary contractor and receives her cost report before the latter communicates with the subcontractor. Therefore, H_1 serves as a natural benchmark establishing what is attainable in a delegation mechanism. This hierarchy provides a good representation of contractual schemes in the construction industry where the customer, first, hires a primary contractor and obtains a cost estimate from her. The primary contractor is then typically given the authority to subcontract other providers whose costs are *ex ante* uncertain.

By the Revelation Principle, the two-agent mechanism is at least as profitable for the principal as H_1. So the questions are whether the principal can achieve the same expected profits in H_1 as in the two-agent mechanism, and how H_1 compares to the single-agent mechanism. An answer to these questions is provided in the following proposition. Before presenting it, let us introduce the following piece of notation. Recall that the quantity schedule in the optimal two-agent mechanism is denoted by $\{q_{LL}^i, q_{LH}^i, q_{HL}^i, q_{HH}^i\}_{i=1}^{i=2}$. Let $\bar{q}_i = \max\{q_{LL}^i, q_{LH}^i\}$ and $\underline{q}_i = \min\{q_{HL}^i, q_{HH}^i\}$.

[11] Typically in this literature, communication costs are not modelled explicitly. Rather, they are assumed to be increasing in the amount of information transmitted between the parties and in the number of rounds of communication. I will use this approach to interpret the results of this section.

Proposition 5. If agent $i \in \{1, 2\}$ serves as the primary contractor, then the principal obtains the same payoff in H_1 as in the two-agent mechanism if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| \leq \frac{1}{1-p_j}$, $i \neq j$, for all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$. Conversely, the hierarchy H_1 with agent $i \in \{1, 2\}$ as the primary contractor is strictly less profitable for the principal than the two-agent mechanism if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| > \frac{1}{1-p_j}$, $i \neq j$, for all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$.

If either agent can serve as the primary contractor, then the principal obtains the same payoff in H_1 as in the two-agent mechanism in the following cases: (i) under complementarity; (ii) under substitutability, if $v_{iii}(.) \geq 0$, $v_{iij} \leq 0$, and $v_{ijj} \geq 0$ for some $i \neq j$ and all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$.

H_1 is strictly more profitable for the principal than the single-agent mechanism if $IC(LL - HH)$ is binding in the latter.

According to Proposition 5, if only one of the agents can serve as the primary contractor, then H_1 is equivalent to the two-agent mechanism when the interdependence between the inputs in their final use is not too large, that is, the marginal benefit/product of one input is not too sensitive to the quantity of the other input.

To understand this result, note that H_1 is equivalent to the two-agent mechanism only if the principal can implement the quantity profile from the optimal two-agent mechanism via H_1 at the same expected cost. It is easy to see that in H_1 each agent obtains at least as much surplus from private information regarding her own cost as in the two-agent mechanism with the same quantity profile. So, H_1 can only attain the same level of profitability as the two-agent mechanism if the primary contractor cannot exploit her role as an informational intermediary to earn additional surplus, and simply passes on the information from the subcontractor to the principal without manipulating it. Manipulating this information could be profitable for the primary contractor for two reasons: (i) she could appropriate part of the informational rent that the principal intends for the subcontractor; (ii) she could extract more surplus from her own information.

In hierarchy H_1, option (i) is infeasible because the primary contractor has to report her cost type before communicating with the subcontractor. Given the primary contractor's report, the informational rents on the subcontractor's information can be appropriated only by the subcontractor. However, option (ii) becomes significant when the report regarding the subcontractor's cost has a large effect on the quantity assigned to the primary contractor, which is exactly when the degree of complementarity or substitutability between the inputs is sufficiently large.

Specifically, suppose that the inputs are complementary and consider the following deviation: the primary contractor misrepresents her low cost as high in the first stage, and then always reports that the subcontractor's cost is low, that is, in states LH and LL, the primary contractor reports state HL. Then, in states LH and LL, the primary contractor has to pay $c_H q^2_{LH}$ to the subcontractor, with a net loss of $\Delta(q^2_{LH} - q^2_{HH})$. However, the expected surplus obtained by the primary contractor on the information about her own cost increases from $\Delta(q^1_{HL}\, p_2 + q^1_{HH}\,(1 - p_2))$ to Δq^1_{HL}. In the proof of Proposition 5, I show that this increase outweighs the extra payment to the subcontractor when the degree of complementarity is sufficiently large. This is so because a report that the subcontractor's cost is low rather than high causes a larger increase in the quantity supplied by the primary contractor, and hence in her informational rent, than in the quantity supplied by the subcontractor, and hence the extra payment to her. Then, in H_1, the principal has to pay a larger informational rent to the primary contractor than in the two-agent mechanism.

Under substitutability, the primary contractor with a low cost has a strong incentive to announce that both costs are high, irrespective of the subcontractor's cost. This incentive is similar to the extra deviation factor and binding incentive constraint $IC(LL - HH)$ in the single-agent mechanism. The principal can offset this incentive to a certain extent by imposing a penalty on the primary contractor when the latter reports that both costs are high. Yet, this penalty cannot be too large, because otherwise the primary contractor will misrepresent her own high cost as low. As a result, the primary contractor's incentive to overstate her cost cannot be mitigated when the degree of substitutability is high, and again the principal has to pay a higher informational rent in H_1.[12]

If the principal can choose either agent to serve as the primary contractor, then under complementarity she can always do so in such a way that H_1 attains the same performance as the two-agent mechanism. This is so because only one of the agents, when serving as the primary contractor, can have an incentive to always report her cost as high and the subcontractor's cost as low.[13] Under substitutability, the ability of the principal to choose either agent to serve as the primary contractor guarantees that H_1 is equivalent to the two-agent mechanism only under additional restrictions on the signs of the third-order derivatives of the benefit function, because both agents could potentially have an incentive to overstate both costs.

Finally, the hierarchy H_1 performs better than the single-agent mechanism if $IC(LL - HH)$ is binding in the latter (which could only happen under substitutability), because in this case the principal can implement the optimal single-agent quantity profile at a lower cost via H_1.

The hierarchy H_1 has two important properties affecting its performance. First, in H_1 the primary contractor's decision whether to report her true cost cannot be contingent on the subcontractor's cost. This reduces the set of feasible deviations by the primary contractor and benefits the principal. Second, only the interim, rather than *ex post*, individual rationality constraints of both agents have to be satisfied in H_1. There is a significant difference between these two types of constraints, in particular, as far as the primary contractor is concerned. With the interim constraints, the principal structures her contract with the primary contractor in such a way that the primary contractor with a high cost obtains a negative payoff for one realization of the subcontractor's cost, and a positive payoff for a different realization of the subcontractor's cost. However, this would be impossible if the primary contractor's *ex post* individual rationality constraint had to be satisfied, as would be the case, for example, if the primary contractor could withdraw from the contract after receiving the subcontractor's cost report.

To understand the significance of these two effects, we will consider three alternative contractual arrangements. First, consider hierarchy H_D in which the primary contractor does not make a cost report to the principal before communicating with the subcontractor. Formally, the sequence of steps in H_D is the same as in H_1, except that stage 3 is eliminated, and in stage 5 the primary contractor reports both costs to the principal. Because the primary contractor accepts the contract with the principal before interacting with the subcontractor, only the interim participation constraints of the primary contractor have to hold. Hierarchy H_D appears to be a good representation of contracting in the defense industry, where marginal costs of production are not learned until significant fixed costs have been incurred, production lines have been built, and supplier relationships have been established.

In H_D, the primary contractor has a larger set of possible deviations than in H_1, as she may decide to misrepresent her cost for one realization of the subcontractor's cost but not for the other realization. Consequently, the primary contractor can try to appropriate some of the informational rents intended by the principal for the subcontractor. In particular, under complementarity, the

[12] Melumad, Mookherjee, and Riechelstein (1995) study H_1 in a model with a continuous distribution of types. They show that H_1 always attains the performance of the two-agent mechanism. The difference in results can be explained as follows. In our discrete model, a minimal deviation in any direction has a finite size equal to the corresponding cost difference. So, a minimal deviation involving a misrepresentation of both costs has a strictly larger size than a minimal deviation which involves misrepresenting only one cost. This size difference in some cases makes the former deviation more attractive than any of the latter. In contrast, in the continuous type model, a minimal deviation in each direction is infinitely small. Therefore, ensuring that incentive constraints hold along each cost dimension separately also ensures that incentive constraints involving a misrepresentation of both costs hold. On a more technical level, it is well known that an agent's informational rent in the continuous multidimensional type model can be computed by integrating the agent's benefit function along any direction from the lowest type (the so-called path independence; see Armstrong, 1996; Krishna and Maenner, 2001; Jehiel, Moldovanu, and Stacchetti, 1999), whereas this is certainly not true in the discrete type model, as incentive constraints do not bind in some directions.

[13] This follows from the first-order conditions (1)–(3) in Lemma 1. Specifically, they imply that if $(q^1_{HL} - q^1_{HH})(1 - p_2) > q^2_{LH} - q^2_{HH}$, then $(q^2_{HL} - q^2_{HH})(1 - p_1) \leq q^1_{LH} - q^1_{HH}$.

primary contractor will have an incentive to misrepresent the state LH as HH in order to reduce the informational rent that she pays to the subcontractor in state LL. As a result, H_D attains the performance of the two-agent mechanism under more restrictive conditions than H_1. Precisely, we have the following.

Proposition 6. If agent $i \in \{1, 2\}$ serves as the primary contractor, then H_D attains the same performance as the two-agent mechanism if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| \le \frac{1-p_i}{1-p_j}$, $j \neq i$, for all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$. Conversely, the hierarchy H_D with agent $i \in \{1, 2\}$ as the primary contractor is strictly less profitable for the principal if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| > \frac{1-p_i}{1-p_j}$, $j \neq i$, for all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$.

If either agent can serve as the primary contractor, then H_D attains the same performance as the two-agent mechanism if $|\frac{v_{12}(q_1,q_2)}{v_{ii}(q_1,q_2)}| \le \frac{1}{1-p_j}$ for each $i \in \{1, 2\}$ *and* $j \neq i$, and all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$.

Comparison of Propositions 5 and 6 shows that the additional deviations available to the primary contractor in hierarchy H_D have real consequences, and in some cases H_1 is strictly more profitable for the principal than H_D. Specifically, under intermediate degrees of complementarity, the principal in H_D has to leave a higher informational rent to the primary contractor to prevent the latter from exaggerating her cost in state LH (without a misreport in state LL). This deviation—unavailable in H_1—allows the primary contractor to reduce the informational rent which she pays to the subcontractor in state LL. Similarly, under intermediate degrees of substitutability, H_D becomes more costly for the principal because she has to prevent the primary contractor from exaggerating her cost only in state LL, with state LH announced truthfully. This deviation is also unavailable in H_1.

Finally, suppose that the primary contractor could opt out of the contract after receiving the subcontractor's report. Then the individual rationality constraints of the primary contractor have to hold *ex post*. Accordingly, let $H_D^{ep}(H_D^{ep})$ be a modification of hierarchy H_1 (H_D) obtained by giving the primary contractor an option to withdraw after receiving the subcontractor's cost report in Stage 5. We then have the following.

Proposition 7. Under substitutability, both H_D^{ep} and H_D^{ep} are strictly less profitable for the principal than the two-agent mechanism.

Under complementarity, we have:

(i) H_D^{ep} attains the same performance as the two-agent mechanism if H_1 attains such performance.
(ii) If agent $i \in \{1, 2\}$ serves as the primary contractor, then H_D^{ep} attains the same performance as the two-agent mechanism if H_D attains the same performance and, additionally, $|\frac{v_{12}(q_1,q_2)}{v_{jj}(q_1,q_2)}| \le \frac{1-p_j}{p_j}$ for all $(q_1, q_2) \in [\underline{q}_1, \bar{q}_1] \times [\underline{q}_2, \bar{q}_2]$, p_j is sufficiently small and p_i is sufficiently large.[14]

In H_D^{ep} and H_D^{ep}, the principal no longer has the freedom to distribute expected payments to the primary contractor across the states of the world in an arbitrary way. This restricts her ability to mitigate the primary contractor's incentives to manipulate the subcontractor's information and/or to capture some of the informational rents intended for the subcontractor. Specifically, because in H_D^{ep} and H_D^{ep} the primary contractor has to earn a nonnegative payoff in state HH, under substitutability the primary contractor has a stronger incentive to report HH in states LH and LL. For this reason, implementation in H_D^{ep} and H_D^{ep} is strictly more costly under substitutability.

Under complementarity, H_D^{ep} performs as well as H_1. But in H_D^{ep} the primary contractor has an even stronger incentive to misrepresent her cost in state LH in order to capture a part of

[14] In the case of a continuous distribution of types, the result that H_D^{ep} does not attain the performance of the two-agent mechanism under substitutability has been established by Melumad, Mookherjee, and Riechelstein (1995), who refer to this hierarchy as H_1'. So the added value of our analysis of this hierarchy lies in deriving the conditions under complementarity when H_D^{ep} attains the performance of the two-agent mechanism.

the informational rent intended for the primary contractor in state *LL*. So, H_D^{ep} attains the same performance as the two-agent mechanism under more restrictive conditions than either H_1 or H_D.

Finally, a few words about the choice of the primary contractor are in order. Propositions 5–7 demonstrate that asymmetries in the cross-effects between the two inputs and differences of cost distributions affect the agents' relative performance as primary contractors. Propositions 5 and 6 show that the principal is better off when the primary contractor is the agent who produces an input that has a smaller effect on the marginal product of the other input and who is more likely to be a high-cost producer. Moreover, the principal benefits when she can choose either agent to serve as the primary contractor. Propositions 5–7 demonstrate that in some cases, the ability to choose the primary contractor ensures that the principal gets the same payoff as in the two-agent mechanism. These results have policy implications for optimal assignment of tasks within hierarchies.

7. Collusion

■ The results of the previous sections can be used to address the issue of collusion in organizations. Laffont and Martimort (1987, 1998)—LM in the sequel—analyze this issue in a similar framework. They consider the same two-agent model as in this article, restricting consideration to the perfect complementarity case that is, when $v(q_1, q_2) = S(\min\{q_1, q_2\})$. So, it is natural to compare the results of this article to theirs and consider how our analysis helps to better understand the effect of collusion.

An opportunity for collusion exists if the agents can communicate with each other and adopt a joint reporting strategy in the mechanism offered by the principal. Formally, the outcome of collusion can be represented by a pair of functions $r(.) : \{L, H\}^2 \mapsto \{L, H\}^2$ and $t^c(.) : \{L, H\}^2 \mapsto \{L, H\}^2$. For every state of the world (i.e., *LL*, *HL*, *LH*, or *HH*), $r(.)$ specifies the state of the world which the agents report in the mechanism and $t^c(.)$ specifies a side transfer from agent 1 to agent 2.

Because each agent has private information about her cost, the collusion game will typically involve some frictions in communication and bargaining between the agents. So, the outcome of the collusion game may have to satisfy certain incentive constraints, and therefore some outcome pairs $(r(.), t^c(.))$ may not be feasible. Which incentive constraints have to hold depends on the specification of the collusion game and the enforceability of collusion.

To avoid model-specific details, in this section I will focus on an important benchmark case of *perfect collusion* in which there is no friction in bargaining between the agents, and any joint reporting strategy $r(.)$ and side transfer function $t^c(.)$ are feasible. In this case, for any mechanism offered by the principal, the agents will choose a joint reporting strategy $r^*(.)$ maximizing the sum of agents' payoffs in each state of the world. We will say that a *stake of perfect collusion* exists in the two-agent mechanism $\{t^1_{KJ}, q^1_{KJ}, t^2_{JK}, q^2_{JK}\}_{K,J\in\{L,H\}}$, if by using such joint reporting strategy $r^*(.)$ the agents can attain a strictly higher sum of payoffs in some state of the world than in this mechanism without collusion.

Clearly, if there is no stake of perfect collusion in a mechanism, then the agents cannot benefit from any collusion game which is less than perfect, that is, when the set of feasible joint reporting strategies is restricted due to informational asymmetry and/or bargaining frictions between the agents. The proof of this assertion is immediate: because perfectly colluding parties can adopt any joint reporting strategy, they can adopt the one that arises as an outcome of any collusion game.

Note that collusion reduces the principal's profits only if a stake of perfect collusion exists in the optimal two-agent mechanism characterized in Section 3. So, it is important to understand the conditions under which such a stake exists. We can answer this question by applying the results of previous sections. Specifically, suppose that the allocation profile and transfers from the optimal two-agent mechanism are assigned in the single-agent mechanism. Then a stake of perfect collusion exists if such mechanism is not incentive compatible, that is, if in some state of the world the single agent earns the highest profit by misrepresenting the costs of both inputs. To understand this, note that perfectly colluding agents maximize the same objective function as the single agent, so they would also be strictly better off by misrepresenting both costs in this state of the world. In the two-agent mechanism, such deviation is not feasible without collusion, but it is feasible under perfect collusion. Using this logic, we can establish that a stake of perfect collusion exists whenever a two-agent mechanism is more profitable for the principal than the single-agent mechanism, and also under substitutability. The latter follows from the fact that under substitutability $q^i_{HL} \geq q^i_{HH}$ for $i \in \{1, 2\}$ in the optimal two-agent mechanism (see Lemma 1). Hence, when this mechanism is assigned to a single agent, the latter would deviate in state *LL* by reporting two high costs.

On the contrary, there is no stake of collusion if the allocation profile from the two-agent mechanism remains incentive compatible in the single-agent mechanism. The proof of Proposition 1 shows that this is the case under the conditions of that proposition. The same argument also works under perfect complementarity. The following proposition summarizes these conclusions.

Proposition 8. A stake of perfect collusion exists in the following two cases: (i) under substitutability; (ii) under complementarity, when the two-agent mechanism is more profitable for the principal than the single-agent mechanism.

A stake of perfect collusion does not exist when the inputs are complementary and $\max\{|\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}|, |\frac{v_{12}(q_1,q_2)}{v_{22}(q_1,q_2)}|\} \leq 1$ for all $q_1, q_2 \in \mathbf{R}^2_+$, and also under perfect complementarity that is, when $v(q_1, q_2) = S(\min\{q_1, q_2\})$, with $S'(.) > 0$ *and* $S''(.) < 0$.

Proposition 8 allows us to understand why LM, who focus on the perfect complementarity case, had to impose additional restrictions on the set of feasible mechanisms in order to generate a stake of collusion. Specifically, they require the principal to offer an anonymous mechanism so that both agents get the same transfer in each state of the world. The anonymity generates a stake of collusion equal to $\Delta(q_{HL} - q_{HH})$ where q_{HL} and q_{HH} are the solutions to the first-order conditions (2)–(5) characterizing the optimal two-agent mechanism in the symmetric case with $p_1 = p_2$ (see Laffont and Martimort, 1998). Obviously, this stake of collusion disappears under substitutability and separability, because in those cases we have $q_{HH} \leq q_{HL}$ (see Lemma 1).

LM (1998) demonstrate that the principal can avoid the cost of preventing collusion in an anonymous mechanism through delegation. Their delegation mechanism (equivalent to our H_D hierarchy) is more profitable for the principal than a two-agent mechanism with collusion. Yet, without an anonymity restriction, this result is not always true. In particular, suppose that inputs are complementary and agent 1 is the primary contractor. Proposition 6 shows that H_D is strictly less profitable than the two-agent mechanism if $|\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}| > \frac{1-p_1}{1-p_2}$, whereas, by Proposition 8, there is no stake of perfect collusion if $\max\{|\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}|, |\frac{v_{12}(q_1,q_2)}{v_{22}(q_1,q_2)}|\} \leq 1$. Both of these conditions hold, and so a nonanonymous two-agent mechanism dominates delegation via H_D hierarchy, for a fairly large class of benefit functions. For example, take a quadratic benefit function with appropriate restrictions on the parameters.

8. Conclusions

■ In this article, I have studied optimal organization of production in an environment where agents have private information about the cost of producing their inputs. The optimality of centralizing production in the hands of a single agent or decentralizing it between two agents depends on whether the value of cost information is sub- or superadditive for the agent(s) which in turn depends on whether the inputs are complementary or substitutable in their final use. Under complementarity or low degrees of substitutability, centralization is optimal, unless the production function is highly asymmetric so that the quantity of one of the inputs affects the marginal benefit of the other input in a significant way. In such case, decentralization is optimal even under complementarity. Decentralization is also optimal when the degree of substitutability is large.

The degree of substitutability/complementarity between the inputs also affects the performance of delegation mechanisms. When it is large, the interdependence between quantities of different inputs is also large in the optimal mechanism. This allows the primary contractor to benefit from her position of an informational intermediary either by increasing the informational rent that she obtains on her cost information or by appropriating part of the informational rent intended for the subcontractor. I have also considered which of the two agents should be chosen as the primary contractor to maximize the performance of a hierarchy. As I have shown, it is optimal to choose the agent who produces an input that has a smaller effect on the marginal product of the other input and who is more likely to be a high-cost producer. The latter result has policy implications for optimal allocation of supervisory functions and assignment of tasks within organizations.

Appendix

■ The following properties of concave functions, established by differentiation, will be useful below.

Property 1. Let $v(.,.)$ be a twice-continuously differentiable, increasing, concave function, and suppose that $v_1(q_1, q_2) = c_1$ and $v_2(q_1, q_2) = c_2$ for some $c_1, c_2 \in (0, \infty)$. Then, $\frac{dq_1}{dc_1} = v_{22} < 0$, $\frac{dq_2}{dc_2} = v_{11} < 0$, $\frac{dq_2}{dc_1} = \frac{dq_1}{dc_2} = -v_{12}$.

Property 2. Suppose that $v_1(q_1, q_2) = c_1$ and $v_2(q_1, q_2) = c_2$ for some $c_1, c_2 > 0$. Then

$$\left|\frac{dq_1}{dc_1}\right| < \left|\frac{dq_2}{dc_1}\right| \quad \text{if} \quad |v_{22}(q_1, q_2)| < |v_{12}(q_1, q_2)|,$$
$$\left|\frac{dq_1}{dc_2}\right| \le \left|\frac{dq_2}{dc_2}\right| \; and \text{and} \; \frac{dq_2}{dc_2} < \frac{dq_2}{dc_1} \quad \text{if} \quad |v_{11}(q_1, q_2)| > |v_{12}(q_1, q_2)|.$$

Proof of Proposition 1. Consider the following single-agent mechanism. In any state of the world KJ, assign the same quantity allocation (q^1_{KJ}, q^2_{JK}) as in the optimal two-agent mechanism, and the corresponding transfer from the following list: $T_{HH} = c_H(q^1_{HH} + q^2_{HH})$, $T_{LH} = c_L q^1_{LH} + c_H q^2_{HL} + \Delta q^1_{HH}$, $T_{HL} = c_H q^1_{HL} + c_L q^2_{LH} + \Delta q^2_{HH}$, $T_{LL} = c_L(q^1_{LL} + q^2_{LL}) + \Delta \max\{q^1_{HL} + q^2_{HH}, q^2_{HL} + q^1_{HH}\}$.

This mechanism is more profitable for the principal than the optimal two-agent mechanism, because her total payment is the same in all states of the world except LL, where her total payment is lower than in the two-agent mechanism by $\Delta \min\{q^1_{HL} - q^1_{HH}, q^2_{HL} - q^2_{HH}\} > 0$. This mechanism satisfies all individual rationality constraints. Let us show that it is incentive compatible. Clearly, it satisfies the downward incentive constraints $IC(LL - HL)$, $IC(LL - LH)$, $IC(LH - HH)$, $IC(HL - HH)$, $IC(LL - HH)$. In particular, the latter holds because $q^i_{HH} < q^i_{HL}$. The upward constraints $IC(HL - LL)$, $IC(LH - LL)$, and $IC(HH - LL)$ hold because $q^i_{KL} > q^i_{KH} <$ by Lemma 1.

Finally, consider the horizontal incentive constraints $IC(LH - HL)$ and $IC(HL - LH)$. Because $IC(LH - HH)$ and $IC(HL - HH)$ are binding, $IC(LH - HL)$ holds if

$$q^2_{LH} - q^1_{HL} \ge q^2_{HH} - q^1_{HH}. \tag{A1}$$

Similarly, $IC(HL - LH)$ holds if $q^1_{LH} - q^2_{HL} \ge q^1_{HH} - q^2_{HH}$. To see that (A1) holds, note that by (1)–(5), $v_2(q^1_{HL}, q^2_{LH}) < v_2(q^1_{HH}, q^2_{HH})$ and $v_1(q^1_{HL}, q^2_{LH}) = v_1(q^1_{HH}, q^2_{HH})$. Because $|v_{11}(q_1, q_2)| \ge v_{12}(q_1, q_2)$, Property 2 implies that (A1) holds. Similarly, the first-order conditions in Lemma 1, the assumption that $|v_{22}(q_1, q_2)| \ge v_{12}(q_1, q_2)$ and, Property 2 imply that $IC(HL - LH)$ holds.

Proof of Proposition 2. Let Condition (6) hold for $i = 2$ and $j = 1$. The proof for $i = 1$ and $j = 2$ is symmetric. To prove the proposition, we compare the profitability of the optimal two-agent mechanism and the relaxed single-agent mechanism $RM(1)$ derived by omitting the incentive constraints $IC(HL - HH)$, $IC(HH - LH)$, and $IC(LL - HH)$ from the principal's profit maximization problem with a single agent. Obviously, $RM(1)$ is more profitable for the principal than the optimal single-agent mechanism, as all constraints have to be imposed in the latter. Hence, if the two-agent mechanism is more profitable than $RM(1)$, then it is also more profitable than the single-agent mechanism.

The proof consists of seven steps. Step 0 is preliminary. In Steps 1–4, I characterize the relaxed single-agent mechanism $RM(1)$. In Step 5, I develop a method for comparing the profitability of $RM(1)$ and the optimal two-agent mechanisms. In Step 6, this method is used to show that the two-agent mechanism is more profitable than $RM(1)$ under Condition (6) of the proposition.

Step 0. Let us show that $2v_{12} + \frac{v_{22}}{1-p_1} + v_{11}(1 - p_1) < 0$. Indeed, consider $\frac{v_{22}}{1-p_1} + v_{11}(1 - p_1)$ as a function of p_1. If $|v_{22}| < |v_{11}|$, then it reaches a unique maximum equal to $-2\sqrt{v_{11}v_{22}}$ at $p_1 = 1 - \sqrt{\frac{v_{22}}{v_{11}}}$. If $|v_{22}| \ge |v_{11}|$, then it reaches a unique maximum equal to $v_{11} + v_{22}$ at $p_1 = 0$. But because $v(.)$ is concave, we have $-2\sqrt{v_{11}v_{22}} + 2v_{12} < 0$. The last inequality implies that $v_{11} + v_{22} + 2v_{12} < 0$.

Further, simple rearrangement shows that with $i = 2$ and $j = 1$, Condition (6) of the proposition can be rewritten as follows:

$$(v_{12}(1-p_1)+v_{22})p_1(1-p_2)+\left(2v_{12}+\frac{v_{22}}{1-p_1}+v_{11}(1-p_1)\right)p_2(1-p_1)>0.$$

Because $2v_{12}+\frac{v_{22}}{1-p_1}+v_{11}(1-p_1)<0$, it follows that $v_{12}+\frac{v_{22}}{1-p_1}>0$.

Step 1. To characterize the relaxed mechanism $RM(1)$, we first solve $RM(1)'$, the principal's profit maximization problem in a single-agent mechanism subject to $IR(HH)$, the individual rationality constraint of HH type, and the downward and horizontal incentive constraints $IC(LL-LH)$, $IC(LL-HL)$, $IC(LH-HH)$, $IC(HL-LH)$, and $IC(LH-HL)$. In Step 4 we show that the solution to $RM(1)'$ satisfies the remaining incentive constraints of $RM(1)$, $IC(HL-LL)$, $IC(LH-LL)$, $IC(HH-HL)$, and $IC(HH-LL)$.

The Lagrangian associated with the problem $RM(1)'$ is

$$\begin{aligned}\max\mathcal{L}&=p_1p_2\left(v\left(g^1_{LL},g^2_{LL}\right)-T_{LL}\right)+p_1(1-p_2)\left(v\left(g^1_{LH},g^2_{HL}\right)-T_{LH}\right)\\&+(1-p_1)p_2\left(v\left(g^1_{HL},g^2_{LH}\right)-T_{HL}\right)+(1-p_1)(1-p_2)\left(v\left(g^1_{HH},g^2_{HH}\right)-T_{HH}\right)\\&+\eta\left(T_{HH}-c_H\left(g^1_{HH}+g^2_{HH}\right)\right)+\lambda_{LH}\left(T_{LL}-c_L(g^1_{LL}+g^2_{LL})-T_{LH}+c_L\left(g^1_{LH}+g^2_{HL}\right)\right)\\&+\lambda_{HL}\left(T_{LL}-c_L\left(g^1_{LL}+g^2_{LL}\right)-T_{HL}+c_L\left(g^1_{HL}+g^2_{LH}\right)\right)+\delta^1_{HH}\Big(T_{LH}-c_Lg^1_{LH}-c_Hg^2_{HL}\\&-T_{HH}+c_Lg^1_{HH}+c_Hg^2_{HH}\Big)+\mu\left(T_{HL}-c_Hg^1_{HL}-c_Lg^2_{LH}-T_{LH}+c_Hg^1_{LH}+c_Lg^2_{HL}\right)\\&+\kappa\left(T_{LH}-c_Lg^1_{LH}-c_Hg^2_{HL}-T_{HL}+c_Lg^1_{HL}+c_Hg^2_{LH}\right).\end{aligned}\tag{A2}$$

The Lagrange multipliers η, λ_{LH}, λ_{HL}, μ, κ, and δ^1_{HH} are nonnegative and satisfy the complementary slackness conditions, $\eta(T_{HH}-c_H(g^1_{HH}+g^2_{HH}))=0$ and similarly for the other constraints. The first-order conditions with respect to transfers are

$$T_{LL}:\quad p_1p_2=\lambda_{LH}+\lambda_{HL}\tag{A3}$$

$$T_{LH}:\quad p_1(1-p_2)=\delta^1_{HH}-\lambda_{LH}-\mu+\kappa\tag{A4}$$

$$T_{HL}:\quad (1-p_1)p_2=-\lambda_{HL}+\mu-\kappa\tag{A5}$$

$$T_{HH}:\quad (1-p_1)(1-p_2)=\eta-\delta^1_{HH}.\tag{A6}$$

The equations (A3)–(A6) imply that $\eta=1$ and $\delta^1_{HH}=p_1(1-p_2)+p_2$, which can be used to simplify the other first-order conditions as follows:

$$v_1\left(g^1_{LL},g^2_{LL}\right)=v_2\left(g^1_{LL},g^2_{LL}\right)=c_L\tag{A7}$$

$$v_1\left(g^1_{LH},g^2_{HL}\right)=c_L-\frac{\mu}{p_1(1-p_2)}\Delta\tag{A8}$$

$$v_2\left(g^1_{HL},g^2_{LH}\right)=c_L-\frac{\kappa}{p_2(1-p_1)}\Delta\tag{A9}$$

$$v_1\left(g^1_{HL},g^2_{LH}\right)=\frac{-\lambda_{HL}+\mu-\kappa}{(1-p_1)p_2}c_H+\frac{\lambda_{HL}+\kappa}{(1-p_1)p_2}\Delta=c_H+\frac{\lambda_{HL}+\kappa}{(1-p_1)p_2}\Delta\tag{A10}$$

$$v_2\left(g^1_{LH},g^2_{HL}\right)=\frac{\delta^1_{HH}-\lambda_{LH}-\mu+\kappa}{p_1(1-p_2)}c_H+\frac{\lambda_{LH}+\mu}{p_1(1-p_2)}\Delta=c_H+\frac{\lambda_{LH}+\mu}{p_1(1-p_2)}\Delta\tag{A11}$$

$$v_1\left(g^1_{HH},g^2_{HH}\right)=c_H+\frac{p_1\Delta}{1-p_1}+\frac{\lambda_{LH}+\mu-\kappa}{(1-p_1)(1-p_2)}\Delta\tag{A12}$$

$$v_2\left(g^1_{HH},g^2_{HH}\right)=c_H+\frac{p_2\Delta}{1-p_2}+\frac{\lambda_{HL}-\mu+\kappa}{(1-p_1)(1-p_2)}\Delta.\tag{A13}$$

Step 2. In this step, we determine which constraints are binding in the relaxed problem (A2) and compute the values of the multipliers. Obviously, the constraint $IR(HH)$ must be binding, because otherwise T_{HH} could be lowered without violating any other constraint. So, $T_{HH}=c_H(g^1_{HH}+g^2_{HH})$. Also, $IC(HL-LH)$ must be binding because otherwise we could lower T_{HL} by a positive amount without violating any incentive constraints.

Next, suppose that constraints $IC(LH-HL)$ and $IC(LL-LH)$ are nonbinding in the solution to problem (A2) (we show this in Step 3). This supposition has several implications: (i) the multipliers κ and λ_{LH} corresponding to these constraints are equal to zero by the complementary slackness condition; (ii) $IC(LH - HH)$ must be binding, because otherwise the principal could lower T_{LH} by a positive amount without violating any incentive constraints; (iii) by (A3) and (A5), $\lambda_{HL} = p_1 p_2$ and $\mu = p_2$, and so $IC(LL - HL)$ is binding. This set of multipliers determines the profile of quantities according to the first-order conditions (A7)–(A13). Because the objective of problem (A2) is concave and all its constraints are linear, this quantity profile constitutes a unique solution to problem (A2), provided that the result of Step 3 holds.

Step 3. In this step, we confirm that $IC(LH - HL)$ and $IC(LL - LH)$ are nonbinding when the Lagrange multipliers take the specified values $\kappa = \lambda_{LH} = 0$, $\lambda_{HL} = p_1 p_2$, $\mu = p_2$.

First, consider $IC(LH - HL)$. It is not binding if

$$T_{LH} - c_L g^1_{LH} - c_H g^2_{HL} > T_{HL} - c_L g^1_{HL} - c_H g^2_{LH}. \tag{A14}$$

By Step 2, the constraint $IC(HL - LH)$ is binding, that is,

$$T_{HL} - c_H g^1_{HL} - c_L g^2_{LH} = T_{LH} - c_H g^1_{LH} - c_L g^2_{HL}. \tag{A15}$$

From (A14) and (A15), it follows that $IC(LH - HL)$ holds and is not binding if $g^1_{LH} - g^1_{HL} + g^2_{LH} - g^2_{HL} > 0$. To establish this inequality, consider the following system:

$$v_1(g^1(t), g^2(t)) = c_H + \Delta\frac{p_1}{1-p_1} - t\Delta\left(1 + \frac{p_1}{1-p_1} + \frac{p_2}{p_1(1-p_2)}\right) \tag{A16}$$

$$v_2(g^1(t), g^2(t)) = c_L + t\Delta\left(1 + \frac{p_2}{p_1(1-p_2)}\right). \tag{A17}$$

Comparing (A16) and (A17) to the first-order conditions (16)–(19), observe that $g^1_{LH} = g^1(1)$, $g^2_{HL} = g^2(1)$, $g^1_{HL} = g^1(0)$, and $g^2_{LH} = g^2(0)$. Hence, $g^1_{LH} - g^1_{HL} + g^2_{LH} - g^2_{HL} = \int_0^1 \frac{dg^1(t)}{dt} - \frac{dg^2(t)}{dt}dt$. Totally differentiating (A16) and (A17) with respect to t and solving for $\frac{dg^1(t)}{dt}$ and $\frac{dg^2(t)}{dt}$, we obtain

$$\begin{aligned}\frac{dg^1(t)}{dt} - \frac{dg^2(t)}{dt} &= \Delta\frac{-v_{22}\left(1+\frac{p_1}{1-p_1}+\frac{p_2}{p_1(1-p_2)}\right) - v_{11}\left(1+\frac{p_2}{p_1(1-p_2)}\right) - v_{12}\left(2+\frac{p_1}{1-p_1}+\frac{2p_2}{p_1(1-p_2)}\right)}{v_{11}v_{22}-v_{12}^2}\\ &= \Delta\frac{\frac{p_2}{p_1(1-p_2)}(-v_{11}-v_{22}-2v_{12}) + p_1(-v_{11}-\frac{v_{12}}{1-p_1}) - \frac{v_{22}}{1-p_1} - v_{11}(1-p_1) - 2v_{12}}{v_{11}v_{22}-v_{12}^2}.\end{aligned} \tag{A18}$$

Let us show that (A18) is strictly positive. Its denominator $v_{22}v_{11} - v_{12}^2$ is positive because $v(.)$ is concave. So, consider its numerator. First, $-v_{11} - v_{22} - 2v_{12} > 0$ by concavity of $v(.)$. Further, $-v_{11} - \frac{v_{12}}{1-p_1} > 0$. This is so because, as shown in Step 0, the condition of the proposition implies that $-\frac{v_{22}}{1-p_1} < v_{12}$ but at the same time $v_{22}v_{11} - v_{12}^2 > 0$ by concavity of $v(.)$. Finally, in Step 0, we have shown that $-\frac{v_{22}}{1-p_1} - v_{11}(1-p_1) - 2v_{12} > 0$. So, $g^1_{LH} - g^1_{HL} + g^2_{LH} - g^2_{HL} > 0$, as required.

Next, consider $IC(LL - LH)$. Because $IC(LL - HL)$ is binding, $IC(LL - LH)$ is nonbinding if

$$T_{LH} - c_L g^1_{LH} - c_L g^2_{HL} < T_{HL} - c_L g^1_{HL} - c_L g^2_{LH}. \tag{A19}$$

Given (A15), (A19) is equivalent to $g^1_{LH} < g^1_{HL}$. Equations (A16) and (A17) yield

$$g^1_{LH} - g^1_{HL} = \int_0^1 \frac{dg^1(t)}{dt} = \Delta\frac{-v_{22}\left(1+\frac{p_1}{1-p_1}+\frac{p_2}{p_1(1-p_2)}\right) - v_{12}\left(1+\frac{p_2}{p_1(1-p_2)}\right)}{v_{11}v_{22}-v_{12}^2}dt < 0.$$

The integrand of this expression is negative for all t because, as shown in Step 0, $-\frac{v_{22}}{1-p_1} < v_{12}$.

Step 4. In this step, we show that the solution to the relaxed problem $RM(1)'$ in (A3) satisfies (as strict inequalities) the omitted upward incentive constraints $IC(HH - HL)$, $IC(HL - LL)$, $IC(LH - LL)$, and $IC(HH - LL)$, all of which are imposed in mechanism $RM(1)$.

First, consider $IC(HH - HL)$. Because $IC(HL - LH)$ and $IC(LH - HH)$ are binding, we have $T_{HL} = c_L g^2_{LH} + c_H g^1_{HL} + \Delta(g^2_{HL} - g^1_{LH} + g^1_{HH})$. So, $IC(HH - HL)$ holds if $g^1_{LH} + g^2_{LH} - g^1_{HH} - g^2_{HL} \geq 0$. In Step 3, we have shown that $g^1_{LH} + g^2_{LH} - g^1_{HL} - g^2_{HL} > 0$. So, $IC(HH - HL)$ holds if $g^1_{HL} > g^1_{HH}$. To see the latter, note that by (A9), (A10), (A12), and (21), we have $v_1(g^1_{HL}, g^2_{LH}) = c_H + \frac{p_1}{1-p_1} < c_H + \frac{p_1}{1-p_1} + \frac{p_2}{(1-p_1)(1-p_2)} = v_1(g^1_{HH}, g^2_{HH})$ and $v_2(g^1_{HL}, g^2_{LH}) = c_L < c_H = v_2(g^1_{HH}, g^2_{HH})$. So, by Property 1, $g^1_{HL} > g^1_{HH}$.

Now, consider $IC(HL - LL)$. Applying Property 1 to two pairs of first-order conditions, the first pair consisting of both equations in (A7) and the second pair consisting of (A8) and (A10), we obtain that $g^1_{LL} > g^1_{HL}$ and $g^2_{LL} > g^2_{LH}$. The inequality $g^1_{LL} > g^1_{HL}$ and the fact that $IC(LL - HL)$ is binding imply that $IC(HL - LL)$ is nonbinding. Similarly, the fact that $IC(LH - HL)$ holds, $IC(LL - HL)$ is binding, and $g^2_{LL} > g^2_{LH}$ imply that $IC(LH - LL)$ holds and is nonbinding.

Finally, consider $IC(HH - LL)$. We have shown that $IC(HH - HL)$ and $IC(HL - LL)$ are nonbinding,

$$T_{HH} - c_H g^1_{HH} - c_H g^2_{HH} > T_{HL} - c_H g^1_{HL} - c_H g^2_{LH} \tag{A20}$$

$$T_{HL} - c_H g^1_{HL} - c_L g^2_{LH} > T_{LL} - c_H g^1_{LL} - c_L g^2_{LL}. \tag{A21}$$

Combining (A20) and (A21) with $g^2_{LL} > g^2_{LH}$ allows us to confirm that $IC(HH - LL)$ holds,

$$T_{HH} - c_H g^1_{HH} - c_H g^2_{HH} > T_{LL} - c_H g^1_{LL} - c_H g^2_{LL}.$$

Step 5. To compare the profitability of the relaxed single-agent mechanism $RM(1)$ and the optimal two-agent mechanism, we connect the principal's maximization problems in the two mechanisms via a homotopy, that is, a continuous transformation.

Homotopy construction. For $t \in [0, 1]$, define $V(t)$ as follows:

$$\begin{aligned} V(t) = \max_h & \left(v\left(h^1_{LL}, h^2_{LL}\right) - c_L\left(h^1_{LL} + h^2_{LL}\right)\right) p_1 p_2 \\ &+ \left(v\left(h^1_{LH}, h^2_{HL}\right) - \left(c_L - \tfrac{\Delta\mu(1-t)}{p_1(1-p_2)}\right) h^1_{LH} - \left[c_H + \Delta\left(\tfrac{p_2 t}{1-p_2} + \tfrac{(\lambda_{LH}+\mu)(1-t)}{p_1(1-p_2)}\right)\right] h^2_{HL}\right) p_1(1-p_2) \\ &+ \left(v\left(h^1_{HL}, h^2_{LH}\right) - \left(c_L - \tfrac{\Delta\kappa(1-t)}{p_2(1-p_1)}\right) h^2_{LH} - \left[c_H + \Delta\left(\tfrac{p_1 t}{1-p_1} + \tfrac{(\lambda_{HL}+\kappa)(1-t)}{p_2(1-p_1)}\right)\right] h^1_{HL}\right)(1-p_1)p_2 \\ &+ \left(v\left(h^1_{HH}, h^2_{HH}\right) - \left[c_H + \Delta\left(\tfrac{p_1}{1-p_1} + \tfrac{(\lambda_{LH}+\mu-\kappa)(1-t)}{(1-p_1)(1-p_2)}\right)\right] h^1_{HH} \right.\\ &\left. - \left[c_H + \Delta\left(\tfrac{p_2}{1-p_2} + \tfrac{(\lambda_{HL}-\mu+\kappa)(1-t)}{(1-p_1)(1-p_2)}\right)\right] h^2_{HH}\right)(1-p_1)(1-p_2). \end{aligned} \tag{A22}$$

For fixed $t \in [0, 1]$, the unique solution $\mathbf{h}^i(t) \equiv (h^i_{LL}(t), h^i_{LH}(t), h^i_{HL}(t), h^i_{HH}(t))$, $(i \in \{1, 2\})$ to the above maximization problem is characterized by the following first-order conditions:

$$v_1\left(h^1_{LL}(t), h^2_{LL}(t)\right) = v_2\left(h^1_{LL}(t), h^2_{LL}(t)\right) = c_L \tag{A23}$$

$$v_1\left(h^1_{LH}(t), h^2_{HL}(t)\right) = c_L - \frac{\Delta\mu(1-t)}{p_1(1-p_2)} \tag{A24}$$

$$v_2\left(h^1_{LH}(t), h^2_{HL}(t)\right) = c_H + \Delta\left(\frac{p_2 t}{1-p_2} + \frac{(\lambda_{LH}+\mu)(1-t)}{p_1(1-p_2)}\right) \tag{A25}$$

$$v_2\left(h^1_{HL}(t), h^2_{LH}(t)\right) = c_L - \frac{\Delta\kappa(1-t)}{p_2(1-p_1)} \tag{A26}$$

$$v_1\left(h^1_{HL}(t), h^2_{LH}(t)\right) = c_H + \Delta\left(\frac{p_1 t}{1-p_1} + \frac{(\lambda_{HL}+\kappa)(1-t)}{p_2(1-p_1)}\right) \tag{A27}$$

$$v_1\left(h^1_{HH}(t), h^2_{HH}(t)\right) = c_H + \Delta\left(\frac{p_1}{1-p_1} + \frac{(\lambda_{LH}+\mu-\kappa)(1-t)}{(1-p_1)(1-p_2)}\right) \tag{A28}$$

$$v_2\left(h^1_{HH}(t), h^2_{HH}(t)\right) = c_H + \Delta\left(\frac{p_2}{1-p_2} + \frac{(\lambda_{HL}-\mu+\kappa)(1-t)}{(1-p_1)(1-p_2)}\right). \tag{A29}$$

Note that $\mathbf{h}^i(0) \equiv \mathbf{g}^i$, $\mathbf{h}^i(1) \equiv \mathbf{q}^i$, and $V(0)$ ($V(1)$) is the principal's expected profit in the mechanism $RM(1)$ (optimal two-agent mechanism). Using the envelope theorem to differentiate $V(t)$, we get

$$\begin{aligned} V(1) - V(0) = \int_0^1 V'(t)dt = \Delta \int_0^1 -\mu h^1_{LH}(t) - \kappa h^2_{LH}(t) + (\lambda_{HL} + \kappa - p_1 p_2) h^1_{HL}(t) dt \\ + \Delta \int_0^1 (\lambda_{LH} + \mu - p_1 p_2) h^2_{HL}(t) + (\lambda_{LH} + \mu - \kappa) h^1_{HH}(t) + (\lambda_{HL} - \mu + \kappa) h^2_{HH}(t) dt. \end{aligned} \tag{A30}$$

Step 6. Let us show that $V(1) > V(0)$. With $\mu = p_2$, $\lambda_{HL} = p_1 p_2$, $\kappa = \lambda_{LH} = 0$, (A30) simplifies to

$$V(1) - V(0) = \Delta \int_0^1 p_2(1-p_1)\left(h^2_{HL}(t) - h^2_{HH}(t)\right) - p_2(h^1_{LH}(t) - h^1_{HH}(t))dt.$$

Observe that $h^1_{LH}(t) - h^1_{HH}(t) = \int_0^1 \frac{\partial h^1(t,s)}{\partial s} ds$ and $h^2_{HL}(t) - h^2_{HH}(t) = \int_0^1 \frac{\partial h^2(t,s)}{\partial s} ds$, where

$$v_1(h^1(t,s), h^2(t,s)) = c_H + \Delta\left(\frac{p_1}{1-p_1} + \frac{(1-t)p_2}{(1-p_1)(1-p_2)}\right) - \frac{\Delta}{1-p_1}\left(1 + \frac{p_2(1-t)}{p_1(1-p_2)}\right)s \tag{A31}$$

$$v_2(h^1(t,s), h^2(t,s)) = c_H + \Delta\frac{p_2 t}{1-p_2} + \Delta\frac{p_2}{p_1(1-p_2)}(1-t)s. \tag{A32}$$

Differentiating (A31) and (A32), we obtain

$$\frac{\partial h^1(t,s)}{\partial s} = \Delta\frac{-v_{22}(h^1(t,s), h^2(t,s))\frac{1}{1-p_1}\left(1 + \frac{p_2(1-t)}{p_1(1-p_2)}\right) - v_{12}(h^1(t,s), h^2(t,s))\frac{p_2}{p_1(1-p_2)}(1-t)}{v_{11}(h^1(t,s), h^2(t,s))v_{22}(h^1(t,s), h^2(t,s)) - v^2_{12}(h^1(t,s), h^2(t,s))} \tag{A33}$$

$$\frac{\partial h^2(t,s)}{\partial s} = \Delta \frac{v_{12}(h^1(t,s),h^2(t,s))\frac{1}{1-p_1}\left(1+\frac{p_2(1-t)}{p_1(1-p_2)}\right)+v_{11}(h^1(t,s),h^2(t,s))\frac{p_2}{p_1(1-p_2)}(1-t)}{v_{11}(h^1(t,s),h^2(t,s))v_{22}(h^1(t,s),h^2(t,s))-v_{12}^2(h^1(t,s),h^2(t,s))}. \tag{A34}$$

Consequently,

$$V(1)-V(0) = \Delta p_2 \int_0^1\int_0^1 \frac{\left(v_{12}(.)+\frac{v_{22}(.)}{1-p_1}\right)+\left(2v_{12}(.)+\frac{v_{22}(.)}{1-p_1}+v_{11}(.)(1-p_1)\right)\frac{p_2(1-t)}{p_1(1-p_2)}}{v_{11}(.)v_{22}(.)-v_{12}^2(.)}dsdt. \tag{A35}$$

Consider the numerator of the integrand of (A35). Step 0 shows that $2v_{12}(.)+\frac{v_{22}(.)}{1-p_1}+v_{11}(.)(1-p_1)<0$. Hence, (A35) is positive if $(v_{12}(.)+\frac{v_{22}(.)}{1-p_1})+(2v_{12}(.)+\frac{v_{22}(.)}{1-p_1}+v_{11}(.)(1-p_1))\frac{p_2}{p_1(1-p_2)}>0$. In Step 0, we have shown that this inequality is equivalent to Condition (6) of the proposition. *Q.E.D.*

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