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Screening when some agents are nonstrategic: does a monopoly need to exclude?

Sergei Severinov*

and

Raymond Deneckere**

We characterize the optimal screening mechanism for a monopolist facing consumers with privately known demands, some of whom have limited abilities to misrepresent their preferences. We show that consumers with better abilities to misrepresent information benefit from the presence of consumers who lack such abilities. Whenever the fraction of the latter group is positive, there is no exclusion: the firm supplies a positive quantity of the good to all consumers whose valuations exceed marginal cost of production. Our analysis is motivated by the evidence indicating that some individuals have limited ability to misrepresent themselves and imitate others.

1. Introduction

■ The nature and qualitative properties of optimal selling strategies for a profit-maximizing monopolist have been explored by many authors. The relevant literature contains detailed analyses of a broad range of selling mechanisms and marketing and pricing schemes, such as different forms of price discrimination, bundling, and tying (see, e.g., Tirole, 1988), and it encompasses a variety of environments. The most ubiquitous situation is one where the monopolist faces a population of heterogeneous consumers with private information about their preferences. The optimal mechanism in this case can be implemented via a simple nonlinear pricing schedule (e.g., Maskin and Riley, 1984). This is the essence of the Taxation Principle.

In practice, however, firms possessing significant market power do not only employ nonlinear pricing, but also rely on direct communication and interaction with customers. Firms in many industries, such as car dealerships, insurance companies, airlines, and publishers, try to elicit information on income, occupation, demographic status, as well as the tastes and habits of their customers, before making a sale to them.

* Duke University; sseverin@duke.edu.

** University of Wisconsin, Madison; rjdeneck@facstaff.wisc.edu.

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The evidence shows that firms use such information—which is clearly related to customers' willingness to pay—in order to offer the same goods or services to different customers at different prices. For example, car salespeople employ various methods and techniques to induce customers to reveal their willingness and ability to pay for the car—which is then used to price an automobile.[1] In Internet commerce, it is becoming common for the prices quoted by Internet stores to depend on the path used to access the site. The path itself, i.e., the history of the customer's visits to the store's and other sites and her responses to questions along the way, contains information about the customer's preferences.

Our article is motivated by these two observations: first, that firms often resort to complicated, costly, and sometimes deliberately nontransparent selling procedures (in the sense that customers are often not informed at the outset about all options that are available to them) designed to extract personal information from customers before making a sale to them; second, that firms are able to use this information to price discriminate and sell the same goods at different prices to different customers. These observations are at odds with the standard approach postulating that all consumers are strategic and able to manipulate their private information in any way they like. Indeed, it would be more cost effective for a firm to avoid building a costly selling mechanism and training its staff in interviewing techniques and instead simply offer a nonlinear pricing schedule—which is known to be optimal in the standard environment. Moreover, selling mechanisms that offer identical products or services at different prices depending on the information provided by the customer would not be feasible in a world populated with standard rational and strategic consumers. Such consumers would infer how their responses affect the price and provide answers signalling that their willingness to pay for the good is low.

We reconcile this apparent discrepancy and explain the aforementioned selling practices and mechanisms by considering an environment where not all agents are strategic and rational in the standard sense. In our economy, some agents have limited cognitive ability, knowledge, or ability to misrepresent their true types, any of which prevents them from imitating the behavior of others in a way that would maximize their payoffs.

There are several reasons to believe that such consumers are present in an economy. At the most basic level, some consumers may not understand whether or how their behavior affects their subsequent terms of trade. One may think about such consumers as naive or boundedly rational.[2] For example, car dealers use a complicated technique called "four square negotiating" to elicit information from less witting customers regarding the level of monthly lease payments that they can sustain.[3]

Secondly, an individual may be unable or unwilling to misrepresent her information if she is naturally averse to lying. For some individuals, the act of lying may be associated with stress or discomfort ("blushing," "feeling wrong"), causing a disutility. This may be due to psychological

[1] The leading marketing textbook on pricing (Nagle, 1987, p. 158) says: "The retail price of an automobile is typically set by the salesperson who evaluates the buyer's willingness to pay. Notice how the salesperson takes a personal interest in the customer, asking what the customer does for a living (ability to pay), how long he has lived in the area (knowledge of the market), what kinds of cars he has bought before (loyalty to a particular brand), and whether he has looked at, or is planning to look at, other cars (awareness of alternatives). By the time a deal has been put together, the experienced salesperson has a fairly good idea how sensitive the buyer's decision will be to the product's price."

[2] We use the terms "bounded rationality" or "naiveté" to describe the following types of behavior. First, a boundedly rational or naive consumer may be unaware of actions that would allow her to conceal her type. Alternatively, such a consumer is unable to understand the surplus extraction motives behind the seller's inquiry about her characteristics. She then reveals her private information truthfully in the belief that this will lead to a closer match between her tastes and the product that she will obtain.

[3] Eskeldson (2000) describes this technique as follows: "A car salesperson will sit you down in front of a blank piece of paper divided into four quadrants. In each quadrant (s)he will fill in values for the price, the trade-in value, the down payment, and the monthly lease rate. The salesperson will then negotiate the four factors separately, crossing out numbers and writing in new ones until the customer is hopelessly confused. The problem is, each of these factors is used to build the monthly payment. By definition, therefore, they cannot be negotiated separately. In the end, you think you have cut a great deal on the price and trade-in, when in fact, all you've done is told the dealer what monthly payment you'll put up with." This technique is so common that apprentice salespersons are taught how to use it during their initial training (see Edmunds, 2003).

or ethical reasons.[4] Erard and Feinstein (1994, p. 2) argue that "some taxpayers appear to be inherently honest, willing to bear their full tax burden even when faced with financial incentives to underreport their income. Evidence for such inherently honest taxpayers ... is supported by econometric evidence and survey findings." Indeed, experimental evidence confirms that a non-negligible portion of the population chooses not to lie regarding private information, even though lying increases their monetary payoffs.[5] Alger and Ma (2003) maintain that some physicians have stronger ethics and are not able to exaggerate the medical problems of a patient when requesting coverage from an HMO, while other physicians are willing to do so. Alger and Renault (2006) investigate the notion of conditional honesty: some agents reveal information truthfully provided they perceive the resulting allocation as sufficiently fair. Alger and Renault (2007) suggest a view of honesty as a precommitment, according to which an honest agent reveals her private information if she has committed to truth-telling *ex ante*. Chen (2000) argues that individuals have a tendency to keep promises, even if it is not always in their self-interest, and shows that this may cause optimal contracts to be incomplete.

Thirdly, some individuals may be unable or find it costly to conceal their personal characteristics when the latter are correlated with observable attributes. For example, an individual's wealth and income level, demographic status, and even preferences can be inferred from observation of that individual's profession, residence, or automobile. Environments where misrepresenting the truth may require costly concealment actions have been studied by Lacker and Weinberg (1989), Maggi and Rodriguez-Clare (1995), and Crocker and Morgan (1998). Lacker and Weinberg (1989, p. 1347) argue that in many instances, "lying about the state of nature requires more than simply sending a false signal regarding one's private information. Often, costly actions must be taken to lend credence to the signals being sent."

Finally, messages may have to be supported by submission of verifiable claims or evidence. For example, telecom firms provide discounts to households that can credibly document their low incomes. Clearly, failure of an individual to produce evidence known to be available to certain types can serve as proof that this individual is of a different type. Then only those with skills and technology to manufacture evidence will be able to mimic others, while those without such technologies will not be able to conceal their private information.[6]

The main goal of our article is to examine how the presence of consumers who have limited ability to misrepresent their private information affects the optimal selling mechanism of a monopolist. In particular, we ask whether and how the monopolist can extract private information from these consumers at little or low cost. The presence of such consumers is incorporated into a standard screening model in the simplest possible way: we assume that a certain fraction of consumers always provides true information about their willingness to pay for the good when asked to report it.

In the context of our model, the reporting of valuations need not be understood literally. It is natural to view it as a reduced form representing the ultimate result of the firm's actions directed at discovering a consumer's willingness to pay (such as interviewing, requesting evidence), as well as the latter's ability or inability to conceal her type. For brevity, consumers who are unable to misrepresent or conceal their private information will be referred to as "honest." However, this term does not pertain exclusively to consumer's ethics. Alternatively, such consumers can

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[4] Multiple studies have documented emotional discomfort that people experience when lying (see, e.g., Ekman, 1973). Note also that all great religions and virtually every human culture we know condemn lying.

[5] Gneezy (2002) reports experiments with deception games in which the proportion of informed senders who chose *not* to mislead opponents—even though misleading was in the senders' best interests—varied from 48% to 83% across experiments. Survey evidence indicates that a core group of people has no qualms at all about inflating claims to insurance companies, but an even greater fraction considers it unacceptable to do so (Tennyson, 1997). Reluctance of individuals to lie may explain why some newspapers and journals provide educational discounts (*Wall Street Journal* and *Financial Times*) or base subscription prices on self-declared income (e.g., the *American Economic Review*) without requiring any verification from customers.

[6] Environments where messages can be supported by "credible claims" have been studied by Lippman and Seppi (1995) and Okuno-Fujiwara, Postlewaite, and Suzumura (1990). Similarly, in Che and Gale (2000) a budget-constrained buyer can credibly disclose information about her budget by posting a bond.

be viewed as boundedly rational or naive. All other consumers can misrepresent their valuations costlessly and will do so to increase their payoffs. Such consumers will be referred to as "strategic." Since a strategic consumer can easily imitate an honest one, honesty or bounded rationality, or naiveté is not an observable characteristic. So, the firm cannot simply segment the market into two parts, i.e., third-degree price discrimination is not feasible.[7]

We derive the optimal selling mechanism for this environment and characterize its properties. Our analysis consists of two parts. First, we derive an optimal game form. Second, we characterize the unique optimal allocation profile implementable via the optimal game form. In the standard environment where all consumers are strategic, the choice of a game form has no real significance. This is implied by the Revelation Principle (or the Taxation Principle). However, the Revelation Principle does not hold in our setup, because the mechanism designer can typically take advantage of the fact that different consumers have different sets of feasible messages by constructing a game form where some types submit nontruthful reports in equilibrium. We establish that the following game form, which we call a "password" mechanism, is optimal in our case. First, a consumer is asked to report her valuation. Then, depending on her report, she is either offered a specific quantity/transfer pair, or is given a menu of quantity/transfer pairs to choose from. The optimality of the password mechanism stems from the fact that an allocation profile implementable via this mechanism has to satisfy a minimal set of incentive constraints. In particular, no incentive constraints of honest consumers have to be satisfied. Using this mechanism, we characterize the optimal allocation profile for an arbitrary fraction of honest consumers in the population. The presence of honest consumers has the following qualitative effects:

(i) Less distortion for the strategic consumers: the quantities assigned to a subset of strategic consumers—in particular, the ones with low valuations—are strictly higher than in the standard "second-best" case with no honest consumers, but still below the first-best. The quantities assigned to the rest of strategic consumer types (in particular, the ones with high valuations) are the same as in the standard case.

(ii) The quantities assigned to a subset of honest consumer types, including the low-valuation types, are below the first-best level but above the quantities assigned to the strategic types with the same valuations, while the quantities assigned to the rest of the honest consumers, including the high-valuation ones, are at the first-best level.

(iii) No exclusion: all consumers whose valuations exceed the marginal cost of production consume a positive quantity, no matter how small the fraction of honest consumers may be.

(iv) Strategic consumers (as well as the firm) benefit from the presence of the honest ones: the surplus earned by every strategic "consumer" type (and the firm's profits) is higher than in the absence of honest consumers. All honest consumers earn zero surplus.

(v) For larger quantities, an honest consumer is charged strictly more than a strategic one. For low quantities, they pay the same amount.

(vi) Over an initial range of quantities, the firm charges quantity premia.

The last result provides an empirically testable implication of our model, since Maskin and

[7] The best example is that of new car sales. Bragg (2004) and Edmunds (2003) describe 'gullible' ('honest') customers who reveal personal information, focus on only one model, and surrender negotiating control to the salesperson, allowing him to bundle price, financing, and trade-in value. These customers typically pay 10–15% above invoice (Bragg, 2004). On the other hand, 'smart' ('strategic') customers investigate promotions and contact several dealerships. They do not divulge personal information, they review the entire line of models, and negotiate only the price of a new car, not trade-in or financing. It is clear, then, how a strategic customer could imitate an honest one. She could understate her income, overstate "family pressure" to be thrifty, feign lack of interest in competing brands and ignorance of promotions. This makes sense if she can convince the salesperson of her limited means, and evidence shows that some customers do buy cars at around invoice (Bragg, 2004). Given the availability of cost and price information, it is puzzling why dealerships using pressure tactics continue to survive. The most natural answer, confirming our model, appears that some customers are boundedly rational, unaware, act on emotion, or have high learning costs.

Riley (1984) have found that the optimal tariff exhibits quantity discounts at all quantity levels in an environment with no honest consumers.

The most surprising qualitative property of the optimal allocation profile is the absence of exclusion. Exclusion is a robust feature of the optimal pricing mechanism in a market with no honest types. Except for the nongeneric case of perfectly inelastic demand at price equal to marginal cost (which requires either that there are no consumers with valuations near marginal cost, or that the density of valuations is infinite at this level), a profit-maximizing monopolist will choose not to sell to consumers whose willingness to pay for the good is not sufficiently higher than marginal cost, under both uniform and nonlinear pricing (see Maskin and Riley (1984) or the discussion in the next section for details). Exclusion must also occur in settings with multidimensional private information (see Armstrong, 1996, and Rochet and Choné, 1998).

If the population consisted only of honest types, then absence of exclusion would be natural. So, intuition based on continuity would suggest that the threshold valuation below which consumers are not served continuously decreases to zero as the fraction of honest types increases. The surprising conclusion of our analysis is that this is not so: as soon as the fraction of honest consumers becomes positive, the threshold level immediately goes down to zero.

The prospect of exclusion is troubling, and may be socially unacceptable, especially when it concerns such vital areas as telecommunication, energy, or transportation. It provides a strong argument in support of government regulation of monopolistic industries. Indeed, it is easy to construct simple examples where the monopolist optimally excludes a significant proportion of consumers.[8] In contrast, our no-exclusion result suggests that such concerns may not be well founded. Even unregulated monopolists will optimally serve all consumers whose willingness to pay for the good is above marginal cost, as long as some consumers in the population do not hide their valuations. Further, if the proportion of honest consumers is nonnegligible (as econometric and experimental evidence suggests), then the consumption of most types who would be excluded in a market without honest consumers is substantial (see Table 1 and Figures 1 and 2).[9]

The contribution of this article can be summarized as follows. First, we explain why firms often resort to selling mechanisms that attempt to directly elicit information from consumers regarding their willingness to pay, instead of using the much simpler and cheaper method of presenting them with nonlinear tariffs. Second, we contribute to the theory of screening by developing a method that can handle a population including both strategic and nonstrategic agents. We use this method to fully characterize an optimal allocation profile for this complex environment. Technically, our contribution lies in introducing new techniques to solve a particular class of multidimensional screening problems. A more general lesson learned from our results is that predictions derived for environments that include only strategic agents may differ qualitatively from predictions for environments in which some nonstrategic agents are present.

The remainder of the article is organized as follows. Section 2 presents the model. Section 3 introduces the main results, and provides intuition. The proofs are relegated to the Appendix and an online supplement available at www.severinov.com/supplement_screen_nonstrategic.pdf.

2. Model and preliminaries

■ A monopoly supplier faces a population of consumers with privately known preferences for the good. Specifically, a consumer with valuation θ gets utility $u(q, \theta) - t$ from consuming quantity q of the good, acquired at cost t. The distribution function $F(\theta)$ of valuations in the population is common knowledge. We assume that $F(\cdot)$ is twice continuously differentiable, and the associated

[8] For example, if a firm with production costs $q^2/2$ faces a population of consumers whose valuation for the good is equal to $\theta \log q$, where θ is consumer's private information and is distributed uniformly on [0, 1], then it is optimal for the firm to exclude all consumers with valuations below 1/2, despite the fact that the good is "essential," i.e., its marginal utility at zero consumption is infinite for all types.

[9] Our results do not rule out the possibility that the government may still want to regulate providers of essential services and make them offer "lifeline" rates to indigent households whose willingness to pay is below marginal cost.

density function $f(\cdot)$ is strictly positive with support consisting of a bounded interval. Without loss of generality, we take this interval to be $[0, 1]$. The consumer's reservation utility level is zero. The firm's cost is additively separable across consumers.[10] We let $c(q)$ denote the cost that the firm incurs when it supplies quantity q to any given consumer.[11]

Equivalently, we can interpret the model as one of quality provision by a monopolist when each consumer has inelastic unit demand. A consumer of type θ gets utility $u(q, \theta)$ from a good of quality q. The firm's marginal cost of producing any unit of quality q is equal to $c(q)$. We maintain the following assumptions on preferences and technology throughout:

Assumption 1. (i) $u(q, \theta)$ is a C^3 function, with $u(q, 0) = 0$, $u(0, \theta) = 0$, $u_q(q, \theta) > 0$, and $u_{\theta q}(q, \theta) > 0$, for all $\theta \in (0, 1]$ and $q \geq 0$.[12]

(ii) $c(q)$ is a C^2 function, with $c(0) = 0$ and $c'(0) = 0$.

(iii) $\exists Q > 0$ s.t. $u(q, \theta) - c(q) < 0$ for all $q > Q$ and $\theta \in [0, 1]$.

(iv) $u(q, \theta) - c(q)$ and $u(q, \theta) - c(q) - [(1 - F(\theta))/f(\theta)]u_\theta(q, \theta)$ are concave in q with strictly negative second derivatives for all $\theta \in [0, 1]$.

Parts (i) and (iv) of Assumption 1 imply that the first-best quantity $q^*(\theta) = \arg\max u(q, \theta) - c(q)$ is unique and increasing in θ.

We modify this standard model by assuming that a fraction $\gamma \in (0, 1)$ of consumers are honest. An honest consumer is not able or not willing to misrepresent (or conceal) her valuation, and truthfully reveals it when asked to report it.[13] The other consumers behave in a standard fashion: they can and will always misrepresent their types if this allows them to obtain a larger surplus. We refer to such consumers as "strategic." Whether a consumer is honest or strategic is not observable, since a strategic consumer can imitate an honest one.

We assume that whether a consumer is honest or strategic is independent of her valuation. This assumption does not qualitatively affect our results and is adopted to simplify the exposition. In the Conclusion, we show how to characterize an optimal allocation profile when the fraction of honest consumers is an arbitrary positive function of the valuation θ.

Our goal is to understand how the presence of honest consumers affects the optimal selling mechanism. The characterization of the optimal mechanism will involve two steps. First, we design an optimal game form for the mechanism. Then we derive an allocation profile that maximizes the firm's expected profits among all allocation profiles that are implementable via the chosen game form.

Let $q(\theta)$ and $g(\theta)$ denote the quantities obtained, respectively, by a strategic and honest consumer with valuation θ, and let $t^s(\theta)$ and $t^\tau(\theta)$ denote the corresponding transfers paid to the firm. An allocation profile is a collection of functions $\{q(\theta), t^s(\theta), g(\theta), t^\tau(\theta)\}$ from $[0, 1]$ into R_+. An allocation profile is implementable via a game form if it is optimal for each consumer type to choose a strategy giving her an allocation corresponding to her true type, i.e., such that a strategic (honest) consumer with valuation θ chooses a strategy that gives her the allocation $q(\theta), t^s(\theta)$ $(g(\theta), t^\tau(\theta))$.

Consider now the choice of a game form. First, note that the Revelation Principle does not hold in our environment. More generally, truthful direct mechanisms may be suboptimal in environments where some agents are not able to misrepresent themselves as certain other types.

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[10] The model can also be interpreted as one of a monopolist supplying a single consumer randomly drawn from a population whose preferences are distributed according to $F(\cdot)$.

[11] A simple fixed-point argument allows us to handle the case in which the monopolist's aggregate cost $C(Q)$ is an increasing function of aggregate output $Q = \int_0^1 q(\theta)f(\theta)d\theta$. Consider the problem with cost function $c(q) = cq$, and let $Q(c)$ denote the aggregate output level selected by the firm. Equilibrium then obtains when $C'(Q(c)) = c$.

[12] We can relax this assumption slightly, to allow for satiation in $u(\cdot)$, as follows. Let $\overline{q}(\theta) = \arg\sup_{q \geq 0} u(q, \theta)$; then $u_q(q, \theta) > 0$ and $u_{q\theta}(q, \theta) > 0$ for all $\theta \in (0, 1]$ and $q \in [0, \overline{q}(\theta))$.

[13] As mentioned above, one can interpret the reporting of valuations as an outcome of a presale interaction between the firm and a customer in which the firm uses different methods and techniques to elicit the customer's willingness to pay for the good and the customer may take steps to misrepresent it.

This result is originally due to Green and Laffont (1986).[14] Intuitively, a mechanism designer can eliminate some incentive constraints by inducing certain types to tell lies that are not feasible for some other types. This allows her to implement a larger set of allocation profiles.

In particular, a mechanism that uses only a nonlinear pricing schedule, or a tariff, is not optimal in our model because a consumer's choice from such a schedule will only depend on her valuation. Hence, the firm would not be able to differentiate honest consumers from strategic ones and exploit the presence of the former. Similarly, a quasi-direct mechanism in which the consumer is only asked to report her valuation and is assigned an allocation based on her report is also suboptimal, because in this mechanism the firm faces a choice between extracting all surplus from honest consumers and making the mechanism incentive compatible for the strategic ones.[15,16]

Generally, a game form is optimal if it allows implementing the largest set of allocation profiles or, equivalently, if in any other game form the set of incentive constraints that have to be imposed on an implementable allocation profile is (weakly) larger. Since a strategic consumer can always imitate any other type, an allocation profile cannot be implemented if it does not satisfy all incentive constraints of strategic consumers. Hence, a game form is optimal if an allocation profile implementable via this game form has to satisfy only the incentive constraints of strategic consumers. This can be achieved via the following "password" mechanism:

Stage 1. A consumer reports her valuation.

Stage 2. (a) If the reported valuation $\widehat{\theta}$ is strictly greater than zero (the lowest valuation), then the consumer is assigned the allocation $g(\widehat{\theta}), t^{\tau}(\widehat{\theta})$.

(b) If the reported valuation is zero (the lowest valuation), then the consumer is given a choice from a menu consisting of $\{q(\theta), t^{s}(\theta)\}_{\theta\in[0,1]}$ and $g(0), t^{\tau}(0)$.

We will say that a mechanism implements an allocation profile *almost everywhere* (a.e.) if the set of types who in this mechanism obtain the allocations corresponding to their true types has full measure. Allocation profiles that differ only on a set of types of measure zero are associated with the same expected profits for the firm. Thus, there is no loss in considering game forms that guarantee implementation only almost everywhere.

Theorem 1. The password mechanism is optimal, i.e., any allocation profile implementable via some game form is also a.e. implementable via the password mechanism.

The password mechanism is optimal because it allows us to ignore all the incentive constraints of honest consumers. Plainly, the report of the lowest-valuation $\theta = 0$ in the first stage can be viewed as a password necessary to access the menu that is designed to be sufficiently attractive and incentive compatible for strategic consumers. Since an honest consumer cannot misrepresent

[14] Deneckere and Severinov (2001) show how one can characterize the set of implementable social choice functions in such environments using an extended Revelation Principle.

[15] If the firm offers an allocation profile that keeps consumers who report truthfully at their reservation utility levels, then it can extract full surplus from honest consumers. However, strategic consumers will then underreport their valuations, reducing the efficiency of the mechanism and the firm's expected profits. Alternatively, the firm can extract a larger surplus from strategic consumers by offering a mechanism where reporting the true valuation is incentive compatible. However, such a mechanism would fail to extract all rents from honest consumers. In an interesting application, Alger and Ma (2003) study how this tradeoff is resolved in the HMO-doctor-patient relationship, with some doctors being more ethical than others and the patients being either healthy or sick.

[16] A direct mechanism in our environment is one where the consumer is asked to report both her valuation and whether she is honest or strategic. Our analysis assumes that honest consumers can claim to be strategic. This assumption is plausible in many contexts. Recall that we use the term honest to describe consumers who are not able to misrepresent their valuation for a variety of reasons, including bounded rationality, naiveté, etc. Yet such consumers may certainly be willing and able to claim that they can act strategically. In this context, direct mechanisms perform no better than the quasi-direct mechanisms, in which consumers are only asked to report their valuations. Alternatively, if honest consumers are unable to claim that they are capable of misrepresentation, the optimal allocation profile in the direct mechanisms would be identical to the one derived in the next section of this article. Thus, our solution is robust to different ways of modelling restrictions on manipulating information by honest types.

her valuation θ, she cannot access the menu if $\theta > 0$. Thus, in the first stage, honest types are effectively separated from strategic ones, and the valuations of honest types are identified.[17] So, the password mechanism allows honest consumers to be distinguished from strategic ones without leaving any surplus to the former, and it also ensures incentive compatibility for the strategic types. It is easy to see that the password mechanism is not a unique optimal game form. However, it minimizes the amount of communication among all optimal game forms (for details, see Deneckere and Severinov (2001).)

Theorem 1 implies that an allocation profile is implementable if and only if it satisfies the incentive constraints of all strategic types, the individual-rationality constraints of all strategic and honest types, and the feasibility constraints $q(\theta) \geq 0$, $g(\theta) \geq 0$ for all $\theta \in [0, 1]$. Let α be a relative proportion of honest to strategic types in the population, i.e.,

$$\alpha = \frac{\gamma}{1-\gamma}.$$

Then we can state the firm's profit-maximization problem as follows:

$$\max_{q(\theta)\geq 0, t^s(\theta), g(\theta)\geq 0, t^\tau(\theta)} \int_0^1 \left(t^s(\theta) - c(q(\theta))\right) f(\theta)d\theta + \alpha \int_0^1 \left(t^\tau(\theta) - c(g(\theta))\right) f(\theta)d\theta \qquad (1)$$

subject to

$$u(q(\theta), \theta) - t^s(\theta) \geq u(q(\theta'), \theta) - t^s(\theta') \qquad \forall\, \theta, \theta' \in [0, 1] \qquad (2)$$

$$u(q(\theta), \theta) - t^s(\theta) \geq u(g(\theta'), \theta) - t^\tau(\theta') \qquad \forall\, \theta, \theta' \in [0, 1] \qquad (3)$$

$$u(q(\theta), \theta) - t^s(\theta) \geq 0 \qquad \forall\, \theta \in [0, 1] \qquad (4)$$

$$u(g(\theta), \theta) - t^\tau(\theta) \geq 0 \qquad \forall\, \theta \in [0, 1]. \qquad (5)$$

The presence of the second set of incentive constraints for strategic consumers, (3), illustrates the multidimensional nature of our problem and explains why the standard approach based on replacing the whole set of incentive constraints with a single differential equation cannot be applied here.

As a first step, we will replace problem (1)–(5) with an equivalent problem that is easier to analyze. For this, we need the following lemma. Recall that $q^*(\theta)$ denotes the first-best quantity.

Lemma 1. If $(q(\cdot), t^s(\cdot), g(\cdot), t^\tau(\cdot))$ is a solution to problem (1)–(5), then (i) $q(\theta)$ is nondecreasing; (ii) $q(0) = 0$ and $q(1) = q^*(1)$; (iii) $g(\theta)$ is nondecreasing and satisfies $g(\theta) \leq q^*(\theta)$; (iv) $t^s(\theta) = u(q(\theta), \theta) - \int_0^\theta u_\theta(q(s), s)ds - U(0)$, where $U(0) = -t^s(0)$; and (v) $t^\tau(\theta) = u(g(\theta), \theta)$.

Notably, part (iv) of Lemma 1 implies that in the optimal mechanism downward incentive constraints between "adjacent" strategic types are binding—as in the standard framework without honest consumers—and the net payoff (informational rent) $U(\theta)$ of a strategic consumer with valuation θ is equal to $U(0) + \int_0^\theta u_\theta(q(s), s)ds$. (Lemma A3 in the Appendix shows that, in fact, it is optimal to set $U(0) = 0$.)

Lemma 1 has several implications. First, we can use (iv) and (v) to substitute $t^s(\theta)$ and $t^\tau(\theta)$ out of (1) and (3), eliminate (2) and (5), and replace (4) with $U(0) \geq 0$. Then integrating (1) by parts (the integration is legitimate because in the optimal mechanism $U(0) \leq u(q^*(1), 1) < \infty$) and imposing the conditions (i)–(iii) of Lemma 1, we obtain the following equivalent to problem (1)–(5):

$$\max_{q(\theta)\geq 0, g(\theta)\geq 0, U(0)\geq 0} -U(0) + \int_0^1 \left(u(q(\theta), \theta) - c(q(\theta)) - u_\theta(q(\theta), \theta)\frac{1-F(\theta)}{f(\theta)}\right) f(\theta)d\theta$$
$$+ \alpha \int_0^1 \left(u(g(\theta), \theta) - c(g(\theta))\right) f(\theta)d\theta \qquad (6)$$

[17] If the seller deals with boundedly rational or naive consumers, it is natural to interpret the password mechanism as follows. The seller offers a mechanism that is complicated and difficult to understand, so that figuring out a method to access the menu would require comprehension and analytical ability that the boundedly rational consumers lack.

subject to

$$\text{(i)}\quad ICT(\theta, \theta')\colon U(\theta) \equiv U(0) + \int_0^{\theta} u_\theta(q(s), s)ds \geq u(g(\theta'), \theta) - u(g(\theta'), \theta')\ \forall\, \theta, \theta' \in [0, 1], \tag{7}$$

(ii) $q(\theta)$ is nondecreasing, (8)

(iii) $g(\theta)$ is nondecreasing and satisfies $g(\theta) \leq q^*(\theta)$ for all $\theta \in [0, 1]$, and (9)

(iv) $q(0) = 0$ and $q(1) = q^*(1)$. (10)

We conclude this section with the following result.

Theorem 2. Existence. Problem (1)–(5) (equivalently, problem (6)–(10)) has a solution in the space of bounded measurable functions.
Uniqueness. If $u_{\theta qq}(q, \theta) \geq 0$ for all $\theta \in [0, 1]$, then the solution is unique.

3. Main results

■ The literature on nonlinear pricing with multidimensional private information (e.g., Wilson (1993); Armstrong, 1996; Rochet and Choné, 1998) points out that identifying the set of binding incentive constraints is the key step toward characterizing the optimal mechanism.

Following this approach in our analysis, we show that in the optimal mechanism either one or two incentive constraints of a strategic consumer are binding. First, by part (iv) of Lemma 1, downward incentive constraints between "adjacent" strategic types are binding—as in the standard framework where all consumers are strategic. Second, for a strategic type with valuation θ, there is exactly one other incentive constraint that may be binding: the incentive constraint between this type and an honest consumer with valuation $r(\theta)$ satisfying $U(\theta) = u(q(\theta), \theta) - u(q(\theta), r(\theta))$.[18] By the single-crossing condition, the latter incentive constraint holds if and only if an honest consumer with valuation $r(\theta)$ is assigned a lower quantity than a strategic consumer with valuation θ.

This implies that $g(\tilde{\theta}) = \min\{q^*(\tilde{\theta}), q(r^{-1}(\tilde{\theta}))\}$, and so the optimal schedule $g(\cdot)$ is uniquely determined by the quantity schedule $q(\cdot)$. Thus, we need to solve only for optimal $q(\cdot)$. This step reduces the dimensionality of the problem and makes it amenable to optimal control methods.

The set of strategic consumer types can be divided into two subsets: the first including types θ for whom the constraint $g(r(\theta)) \leq q(\theta)$ is binding (case 1), and the second including types for whom this constraint is nonbinding (case 2). Lemma A9 (in the Appendix) establishes that both sets are nonempty, with case 1 applying for low θ's and case 2 applying for high θ's. Furthermore, Lemma A10 (in the Appendix) establishes that under plausible conditions on the utility function, there is a unique switchpoint between the intervals where cases 1 and 2 apply.

The nature of the solution for $q(\cdot)$ at a particular point depends critically on whether case 1 or case 2 applies there. Intuitively, the optimal $q(\cdot)$ in both cases is determined by the well-known tradeoff between efficiency and informational rents. However, when case 1 applies, this tradeoff is qualitatively different from the standard case without honest consumers. When case 2 applies, this tradeoff is similar to the standard case.

At first, let us focus on case 1. Consider a strategic consumer with valuation θ that belongs to this case. In order to reduce this type's informational rent $U(\theta)$, the firm will need to reduce not only the quantities assigned to strategic consumers with valuations less than θ (as in the standard case), it will also have to reduce the quantity $g(r(\theta))$ assigned to the honest consumer with valuation $r(\theta)$ (who gets zero surplus because information regarding her valuation is elicited for free).

Therefore, in case 1 the efficiency losses from quantity reductions are exacerbated by the fact that the firm has to reduce the quantities assigned both to strategic and honest consumers. As a result, the tradeoff between the efficiency of an allocation profile and informational rents

[18] In words, the valuation $r(\theta)$ is defined via the following condition. If an honest consumer with valuation $r(\theta)$ was assigned the same quantity $q(\theta)$ that is assigned to the strategic consumer with valuation θ and hence would have to pay $u(q(\theta), r(\theta))$, then the strategic consumer with valuation θ would be indifferent between the allocation designed for her and the allocation designed for the honest consumer with valuation $r(\theta)$.

has to shift toward higher efficiency, in comparison to the standard environment with no honest consumers. Indeed, Lemma A5 (in the Appendix) demonstrates that $q(\theta)$ is closer to the efficient (first-best) quantity level than the optimal quantity in the standard environment, and $g(\cdot)$ is even closer to the first-best than $q(\cdot)$. Yet the downward distortions do not disappear. Specifically, in case 1, $q^{sb}(\theta) \leq q(\theta) < g(\theta) \leq q^*(\theta)$, where $q^{sb}(\theta)$ denotes the optimal quantity in the standard model without honest consumers. Also note that strategic and honest types pay the same amount for the same quantity when case 1 applies. This follows immediately from the fact that in case 1 the incentive constraint between strategic consumer with valuation θ and honest consumer with valuation $r(\theta)$ is binding and both of them obtain the same quantity $q(\theta)$.

An important manifestation of the shift in the tradeoff between efficiency and informational rents is the following no-exclusion result.

Theorem 3. For any $\alpha > 0$, the optimal allocation profile $q(\theta), g(\theta), t^s(\theta), t^\tau(\theta)$ is such that $q(\theta) > 0$ and $g(\theta) > 0$ for all $\theta \in (0, 1]$.

This result stands in contrast with the standard model without honest consumers where the optimality of exclusion is a very robust property. In fact, under our maintained assumptions, the optimal quantity schedule exhibits exclusion when the fraction of honest consumers in the population is zero.[19] As shown by Armstrong (1996), exclusion is also generic in the multidimensional nonlinear pricing model.

The intuition behind the optimality of exclusion is well understood. Starting from a tariff under which all customers participate in the market, if the monopolist introduces a small fixed charge ε, she will gain ε from every customer remaining in the market, but lose any amount she collected on customers who leave the market. In the regular case, both the number of lost customers and the revenue per lost customer are of order ε, so total losses are an order of magnitude smaller than total gains, and exclusion pays. Only if there are no low-valuation customers (so that the tariff can be set sufficiently high without exclusion) or if the density of low-valuation customers is infinite (so that the revenue lost by excluding them is large) may exclusion fail to occur.[20]

Absence of exclusion in our mechanism is explained by two factors. First, in Lemma A4 (in the Appendix) we establish the following "common cutoff" property that follows from a simple analysis of incentive compatibility: if strategic consumers with valuations below some threshold level $\underline{\theta} > 0$ are assigned zero quantity, then so are the honest consumers with valuations below $\underline{\theta}$. Second, since the firm does not leave any surplus to the honest consumers, rationing them is costly. Particularly, if the firm raises $q(\theta)$ on the interval $[\underline{\theta}/2, \underline{\theta}]$ to some small $\varepsilon > 0$, then the profits that it collects from strategic consumers decrease. But the firm can now assign positive quantities to honest consumers with valuations in $[\underline{\theta}/2, \underline{\theta}]$ and collect payments from them. Theorem 3 is proven by showing that the extra profits collected from the honest consumers is of a higher order of magnitude than the loss of profits from the strategic consumers. Since this is true for arbitrary $\underline{\theta} > 0$, there can be no exclusion in the optimal mechanism.

The intuition for the difference in the order of magnitudes is as follows. When the firm raises $q(\theta)$ on the interval $[\underline{\theta}/2, \underline{\theta}]$ from zero to ε, all strategic consumers obtain higher informational rents $U(\cdot)$. The magnitude of the associated decrease in the firm's expected profits is of order ε. Yet the firm can now assign positive quantities $g(\cdot)$ to honest consumers with valuations in $[\underline{\theta}/2, \underline{\theta}]$. Specifically, $g(\cdot)$ can be set at the level where the incentive constraints between some strategic consumers and honest consumers with valuations in $[\underline{\theta}/2, \underline{\theta}]$ become binding, i.e., $U(r^{-1}(\theta)) = u(g(\theta), r^{-1}(\theta)) - u(g(\theta), \theta)$, where $r^{-1}(\theta)$ is the valuation of a strategic consumer

[19] Consider virtual surplus $u(q, \theta) - c(q) - [(1 - F(\theta))/f(\theta)]u_\theta(q, \theta)$. Since $u_{q\theta}(0, \theta)$ is bounded away from zero, and since $u_q(0, \theta) - c'(0)$ converges to zero as $\theta \longrightarrow 0$, virtual surplus will be maximized at $q = 0$ for a nondegenerate interval of types containing $\theta = 0$. More generally, exclusion will be present provided $f(0) < \infty$ and $(u(q, \theta) - c(q))/u_\theta(q, \theta) \longrightarrow 0$ as $\theta \longrightarrow 0$.

[20] Let $q^{sb}(\theta)$ denote the standard second-best quantity schedule in the model without honest consumers. While $q^{sb}(\cdot)$ exhibits a range of exclusion and $q(\cdot)$ does not, it is nevertheless the case that $q(\cdot)$ converges to $q^{sb}(\cdot)$ as $\alpha \longrightarrow 0$. More precisely, $\max_\theta (q(\theta) - q^{sb}(\theta)) \longrightarrow 0$ as $\alpha \longrightarrow 0$. This property is illustrated in Figure 2.

who is indifferent between the allocation designed for her and that of the honest consumer with valuation θ. This equation implies that $g(\theta)$ is of order $U(r^{-1}(\theta))/(r^{-1}(\theta) - \theta)$.[21]

In the limit as ε converges to zero, only strategic types with valuations close to $\underline{\theta}$ are willing to imitate honest types with valuations close to $\underline{\theta}$. This is so because strategic consumers with valuations strictly above $\underline{\theta}$ earned strictly positive rents prior to the described modification, and therefore they do not find it attractive to imitate honest types who are assigned low quantities and do not earn any informational rents. Hence, for sufficiently small ε, $r^{-1}(\theta)$ is close to $\underline{\theta}$ for all $\theta \in [\underline{\theta}/2, \underline{\theta}]$. Consequently, since $U(\underline{\theta})$ is of order $\varepsilon\underline{\theta}$ for small ε,[22] $g(\theta)$ is of order $\varepsilon[\underline{\theta}/(\underline{\theta} - \theta)]$. So, $g(\theta)$ is an order of magnitude larger than ε for θ near $\underline{\theta}$.

Finally, since the monopolist extracts full surplus from honest consumers, its payoff from each honest consumer type is of the same order of magnitude as the quantity assigned to her. This implies that near $\varepsilon = 0$, the profits gained from honest consumers with valuations in $[\underline{\theta}/2, \underline{\theta}]$ are much larger then the loss incurred by leaving higher informational rents to strategic consumers.

Let us now turn to case 2, where incentive constraints between strategic and honest consumers are not binding. We establish that in this case, $q(\theta) = q^{sb}(\theta)$, i.e., the quantity assignment for strategic consumers is the same as in the standard environment (see Lemma A9 in the Appendix), because the tradeoff between efficiency and informational rent is also the same. As mentioned above, case 2 applies for high θ. The intuition for this is as follows. When θ is high, the informational rent $U(\theta)$ of a strategic consumer is sufficiently large that it exceeds the payoff she could get by imitating an honest consumer with valuation $r(\theta)$, even if the latter was assigned the first-best quantity.

Although $q(\theta) = q^{sb}(\theta)$ in case 2, there is a sort of "domino effect" in the informational rents of strategic consumers. To prevent strategic consumers with low valuations from imitating honest types, they are assigned larger quantities and thus earn higher informational rents. As a consequence, all strategic types obtain higher surpluses than in the standard case. Strategic consumers therefore benefit from the presence of honest ones. The former are paid more to prevent them from imitating the latter.

Importantly, since case 2—where incentive constraints between strategic and honest are not binding—applies for high θ's, and high-valuation honest and strategic customers consume quantities in the same range, it follows that honest customers pay higher prices for the same quantities in the high range. Note, however, that honest and strategic consumers with the same valuation consume different quantities, except for the highest-valuation type.

The following theorem gives a precise characterization of the optimal quantity schedule $q(\theta)$ and $r(\theta)$ under an assumption guaranteeing that the monotonicity constraint $q'(\theta) \geq 0$ is nonbinding.

Theorem 4. If $F(\theta) + f(\theta)[(u_q(q, \theta) - c'(q))/u_{\theta q}(\theta, q)]$ is increasing in θ for all $\theta \in [0, 1]$ and $q \in [0, q^*(1)]$,[23] then the optimal quantity schedule $q(\theta)$ and $r(\theta)$ solve the following system of differential equations[24] with boundary conditions $q(0) = r(0) = 0$ and $q(1) = q^*(1)$:

$$q'\left(\alpha \max\left\{f(r)\frac{u_q(q, r) - c'(q)}{u_\theta(q, r)}, 0\right\} + f(\theta)\frac{(u_q(q, \theta) - c'(q))u_{\theta qq}(q, \theta)}{u_{\theta q}(q, \theta)^2} - f(\theta)\frac{u_{qq}(q, \theta) - c''(q)}{u_{\theta q}(q, \theta)}\right)$$

$$= 2f(\theta) + f'(\theta)\frac{u_q(q, \theta) - c'(q)}{u_{\theta q}(q, \theta)} - f(\theta)\frac{(u_q(q, \theta) - c'(q))u_{\theta\theta q}(q, \theta)}{u_{\theta q}(q, \theta)^2} \tag{11}$$

$$r'(\theta) = q'\frac{u_q(q, \theta) - u_q(q, r)}{u_\theta(q, r)}. \tag{12}$$

[21] This is most easily seen when $u(q, \theta) = q\theta$, for then $u(g(\theta), r^{-1}(\theta)) - u(g(\theta), \theta) = g(\theta)(r^{-1}(\theta) - \theta)$.

[22] Again, if $u(q, \theta) = q\theta$, then $U(\underline{\theta}) = \varepsilon\underline{\theta}$ precisely.

[23] Note that this assumption is "standard": it is implied by the ones that are normally used to ensure that the quantity monotonicity constraint is not binding in the model with $\alpha = 0$. See Fudenberg and Tirole (1991).

[24] In the Conclusion we exhibit the counterpart of (11) for the case when α (or, equivalently, γ) varies with θ.

Furthermore, if $f(\theta)[(u_q(q,\theta)-c'(q))/u_{\theta q}(\theta,q)]$ is increasing in θ, for all $\theta \in [0,1]$ and $q \in [0, q^*(1)]$, then there is exactly one such solution.

Several comments are in order here. First, the term $\max\{f(r)[(u_q(q,r)-c'(q))/u_\theta(q,r)],0\}$ on the left-hand side of (11) highlights the distinction between cases 1 and 2. In case 1 the constraint $g(r(\theta)) \le q^*(\theta)$ is binding, and so $u_q(q,r)-c'(q) > 0$. In contrast, in case 2 this constraint is not binding, and so $u_q(q,r)-c'(q) \le 0$. Thus, the term $\max\{f(r)[(u_q(q,r)-c'(q))/u_\theta(q,r)], 0\}$ is positive in case 1 and zero in case 2. So, one can say that the slope of $q(\theta)$ is flatter in case 1 than in case 2. Figure 4 depicts the solution when there is a unique switchpoint between cases 1 and 2 (which is so under the conditions of Lemma A10 in the Appendix). The valuation of a strategic consumer lying on the boundary between the two cases is denoted by $\overline{\theta}$.

Although according to Lemma A1 in the Appendix the optimal allocation profile is unique when $u_{\theta qq}(q,\theta) \ge 0$, the singularity of (11) at zero[25] implies that the system (11) and (12) can potentially have several solutions satisfying the boundary conditions $q(0)=r(0)=0, q(1)=q^*(1)$. A condition that rules out this possibility is that $f(\theta)[(u_q(q,\theta)-c'(q))/u_\theta(q,\theta)]$ is increasing in θ (see Lemma A11 in the Appendix). If this condition is not met and there are, indeed, multiple solutions to (11)–(12), one would have to choose the optimal one among them by comparing the corresponding values of the firm's expected profits.

The condition that $F(\theta)+f(\theta)[(u_q(q,\theta)-c'(q))/u_{\theta q}(\theta,q)]$ is increasing guarantees that the constraint requiring $q(\theta)$ to be nondecreasing does not bind. Indeed, this condition implies that the right-hand side of (11) is nonnegative. Meanwhile, Assumption 1(iv) ensures that the left-hand side is positive.[26] If $F(\theta)+f(\theta)[(u_q(q,\theta)-c'(q))/u_{\theta q}(\theta,q)]$ is not always increasing in θ, the constraint $q' \ge 0$ may become binding. In this case, the solution will contain intervals on which $q(\theta)$ is strictly increasing and intervals on which it is constant ("ironed"). On the former intervals the solution will still be characterized by (11). The intervals on which $q(\cdot)$ is constant are determined by using an "ironing" procedure similar to the one in Guesnerie and Laffont (1984) but modified due to the special nature of our problem with honest consumers. The details of our ironing procedure are described in the Appendix.

Next, we characterize the solution explicitly in a special but common case.

Corollary 1.[27] Suppose that $u(q,\theta)=\theta q - q^2/2, c(q)=0$, and $F(\cdot)$ is uniform.[28] Then there is a unique optimal allocation profile $(q(\theta), g(\theta), t^s(\theta), t^\tau(\theta))$ maximizing the monopolist's expected profits. Both $q(\theta)$ and $g(\theta)$ are strictly increasing and continuous, and $q(\theta)$ is convex on $[0,1]$.

The transfers satisfy $t^s(\theta)=\theta q(\theta)-\int_0^1 q(s)ds$ and $t^\tau(\theta)=\theta g(\theta)$.

If $\alpha \neq 4$, then for all $\theta \in [0, 2/3+[1/(3(\sqrt{1+2\alpha}+1))]$, $q(\theta)$ is the unique solution in the range $[0, 1/3+2/(3(\sqrt{1+2\alpha}+1))]$ satisfying the following equation:

$$\frac{1-\alpha}{2-\alpha/2}q(\theta)+\frac{\left(1+\dfrac{\alpha}{2(\sqrt{1+2\alpha}+1)}\right)}{(2-\alpha/2)\left(\dfrac{1}{3}+\dfrac{2}{3(\sqrt{1+2\alpha}+1)}\right)^{(\sqrt{1+2\alpha}-1)/2}}q(\theta)^{(\sqrt{1+2\alpha}-1)/2}=\theta. \quad (13)$$

[25] This singularity implies that it may not be easy to solve the system (11)–(12) starting at the origin, even numerically. It could be easier then to solve the system in reverse, by starting at $\theta=1$. However, the boundary value $r(1)$ is not *a priori* given. One recipe would be to derive solutions to (11) and (12) starting from different values of $r(1)$ and then choose the one(s) that also satisfy $q(0)=r(0)=0$.

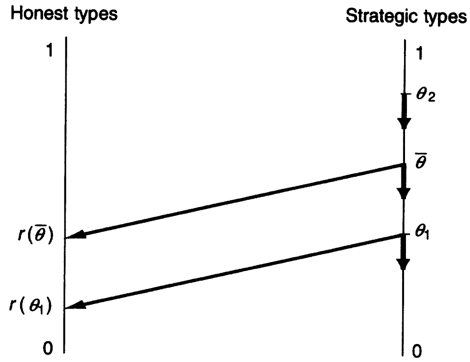
[26] If $u_{qq\theta}(q,\theta) \ge 0$, then the concavity of $u(q,\theta)-c(q)$ in q and the fact that $q(\theta) \le q^*(\theta)$ (see Lemma A5) imply that $[(u_q(q,\theta)-c'(q))/u_{\theta q}(\theta,q)]u_{qq\theta}(q,\theta)-[u_{qq}(q,\theta)-c''(q)] > 0$. Suppose, on the other hand, that $u_{qq\theta}(q,\theta)<0$. By Lemma A5, $q^{sb}(\theta) \le q(\theta)$, so $u_q(q,\theta)-c'(q)-[(1-F(\theta))/f(\theta)]u_{q\theta}(q,\theta) \le 0$. It follows that $[(u_q(q,\theta)-c'(q))/u_{\theta q}(\theta,q)]u_{qq\theta}(q,\theta)-(u_{qq}(q,\theta)-c''(q)) \ge -(u_{qq}(q,\theta)-c''(q)+[(1-F(\theta))/f(\theta)]u_{qq\theta}(q,\theta)) > 0$, where the last inequality holds because the function $u(q,\theta)-c(q)-[(1-F(\theta))/f(\theta)]u_\theta(q,\theta)$ is concave in q.

[27] The proofs of Corollaries 1 and 2 are available at www.severinov.com/supplement_screen_nonstrategic.pdf.

[28] Alternatively, we can take $u(q,\theta)=\theta q$ and $c(q)=q^2/2$. The optimal quantity allocations are the same under these two formulations. Also, instead of $c(q)=0$ we could choose $c(q)=\bar{c}q$ for some $\bar{c}>0$. In this case all types with valuations below $\bar{c}$ would be assigned zero quantities and any type with valuation $\theta > \bar{c}$ would get the same allocation as the type with valuation $(\theta-\bar{c})/(1-\bar{c})$ in the original model.

FIGURE 1

BINDING INCENTIVE CONSTRAINTS IN THE OPTIMAL MECHANISM



For all $\theta \in [2/3 + 1/(3(\sqrt{1+2\alpha}+1)), 1]$, we have $q(\theta) = 2\theta - 1$.

For all $\theta \in [0, 1/3 + 2/(3(\sqrt{1+2\alpha}+1))]$, $g(\theta)$ is the unique solution in the range $[0, 1/3 + 2/(3(\sqrt{1+2\alpha}+1))]$ satisfying the following equation:

$$\frac{1-\alpha}{4-\alpha} g(\theta) + \frac{3^{(\sqrt{1+2\alpha}-1)/2}}{(4-\alpha)} \left(\frac{\sqrt{1+2\alpha}+1}{\sqrt{1+2\alpha}+3}\right)^{(\sqrt{1+2\alpha}-3)/2} g(\theta)^{(\sqrt{1+2\alpha}-1)/2} = \theta. \tag{14}$$

For all $\theta \in [1/3 + 2/(3(\sqrt{1+2\alpha}+1)), 1]$, we have $g(\theta) = \theta$.[29]

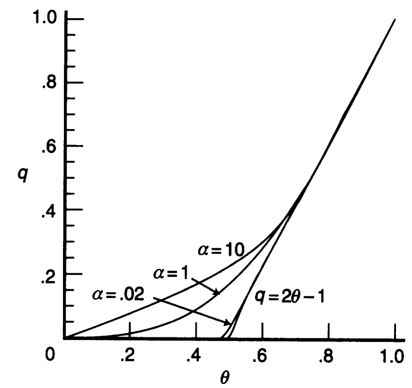
In the proof, we demonstrate that (13) and (14) are invertible over their respective ranges, and so $q(\theta)$ and $g(\theta)$ are well defined.

The two-part nature of the optimal schedules $q(\theta)$ and $g(\theta)$ derives from the fact that the incentive constraint between a strategic consumer with valuation θ and an honest consumer with valuation $r(\theta)$ is binding when $\theta \in [0, \bar{\theta}]$ (case 1 applies), and is not binding if θ belongs to the interval $(\bar{\theta}, 1]$ (case 2 applies) where $\bar{\theta} = 2/3 + 1/(3(\sqrt{1+2\alpha}+1))$. The structure of binding incentive constraints is depicted in Figure 1. Consistently with Lemma A5, $q^{sb}(\theta) \le q(\theta) \le q^*(\theta)$ for all $\theta \in (0, \bar{\theta})$, $q(\theta) = q^{sb}(\theta)$ for all $\theta \in [\bar{\theta}, 1]$, while $g(\theta) = q^*(\theta) = \theta$ for all $\theta \in [r(\bar{\theta}), 1]$.

Figures 2 and 3 illustrate the solution for three different values of α: 10, 1, and .02. When

FIGURE 2

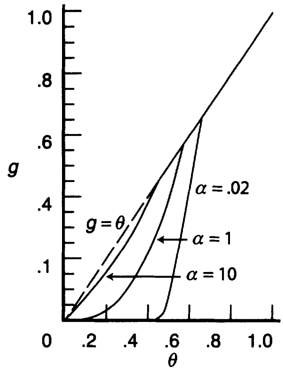
OPTIMAL QUANTITY SCHEDULE $q(\cdot)$ OF THE STRATEGIC CONSUMERS



[29] If $\alpha = 4$, then the equation characterizing $q(\theta)$ on the lower part of the type space $[0, 3/4]$ is $(3/2 - \log(2q))q = \theta$, while the equation characterizing $g(\theta)$ on $[0, 1/2]$ is $(1 - \log(2g)/2)g = \theta$. They can be derived either directly or by taking the appropriate limits of the equations characterizing the solution for $\alpha \neq 4$.

FIGURE 3

OPTIMAL QUANTITY SCHEDULE $g(\cdot)$ OF THE HONEST CONSUMERS



$\alpha = 1$, i.e., half of the population is honest and the other half is strategic, the optimal quantity schedules have the following closed form:

$$q(\theta) = \begin{cases} \sqrt{3}^{\sqrt{3}}(\sqrt{3}-1)^{\sqrt{3}+1}\theta^{\sqrt{3}+1} & \text{if } \theta \in \left[0, \dfrac{1}{3-\sqrt{3}}\right] \\ 2\theta - 1 & \text{if } \theta \in \left[\dfrac{1}{3-\sqrt{3}}, 1\right] \end{cases}$$

$$g(\theta) = \begin{cases} \sqrt{3}^{\sqrt{3}}\theta^{\sqrt{3}+1}\left(= q\left(\dfrac{\theta}{\sqrt{3}-1}\right)\right) & \text{if } \theta \in \left[0, \dfrac{1}{\sqrt{3}}\right] \\ \theta & \text{if } \theta \in \left[\dfrac{1}{\sqrt{3}}, 1\right]. \end{cases}$$

By varying α, the ratio of honest consumers to strategic ones in the population, we establish a number of interesting comparative-statics results described in the corollary below. Let us explicitly incorporate the dependence of the solution on α by using the notation $q(\cdot, \alpha)$, $g(\cdot, \alpha)$, $U(\cdot, \alpha)$, and $\overline{\theta}(\alpha)$ for the quantity schedules, the informational rent, and the threshold value between cases 1 and 2, respectively.

Corollary 2. Suppose that $u(q, \theta) = \theta q - q^2/2$, $c(q) = 0$ and $F(\cdot)$ is uniform. Let α_1 and α_2 be such that $\alpha_1 > \alpha_2 > 0$. Then,

(i) There exists a unique $\theta_c \in (0, 2/3 + 1/(3(\sqrt{1+2\alpha_1}+1)))$ s.t. $q(\theta_c, \alpha_1) = q(\theta_c, \alpha_2)$. Furthermore, $q(\theta, \alpha_1) > q(\theta, \alpha_2)$ for $\theta \in (0, \theta_c)$, and $q(\theta, \alpha_1) < q(\theta, \alpha_2)$ for $\theta \in (\theta_c, 2/3 + 1/(3(\sqrt{1+2\alpha_2}+1)))$.

(ii) $U(\theta, \alpha_1) > U(\theta, \alpha_2)$ for $\theta \in (0, 1]$.

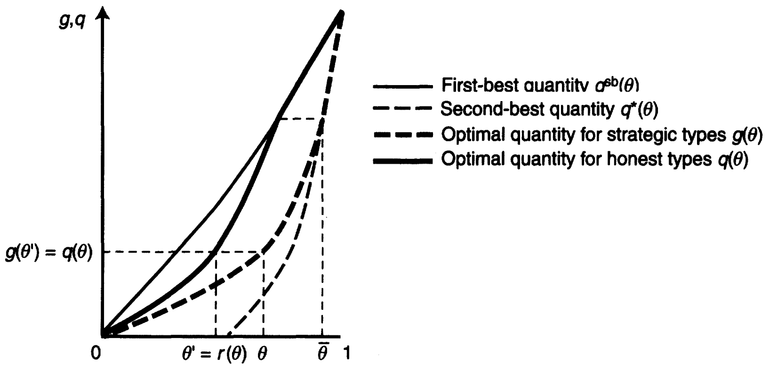
(iii) $g(\theta, \alpha_1) > g(\theta, \alpha_2)$ for $\theta \in (0, 1/3 + 2/(3(\sqrt{1+2\alpha_2}+1)))$.

Parts (ii) and (iii) are easy to understand. Clearly, as the proportion of honest consumers increases, the benefit to the firm from increasing $g(\cdot)$ toward an efficient level and extracting more surplus from the honest consumers becomes larger than the cost of an associated increase in informational rents $U(\cdot)$ paid to strategic consumers whose fraction has now decreased.

The explanation for part (i) is similar but more complex. Intuitively, an increase in the proportion of honest consumers causes a shift toward higher efficiency of the allocation profile. As a result, $q(\theta, \alpha)$ increases in α. However, this is only the case for small θ, as the effect disappears when θ is high. This happens because the informational rent $U(\theta, \alpha)$ of a strategic consumer with valuation $\theta > \overline{\theta}(\alpha)$ is sufficiently large that she has no incentive to imitate any honest consumer, even if honest consumers with valuations exceeding $r(\overline{\theta}(\alpha), \alpha)$ are assigned efficient quantities.

FIGURE 4

OPTIMAL QUANTITY SCHEDULE WITH A UNIQUE SWITCHPOINT $\bar{\theta}$



Therefore, if $\theta > \bar{\theta}(\alpha)$, it is no longer optimal for the firm to raise $q(\theta, \alpha)$ above the quantity optimal in the standard case. So the downward distortions in the quantity schedule for strategic consumers persist even as the fraction of honest consumers converges to one.

Since the firm can set $g(\cdot, \alpha)$ efficiently on the interval $[r(\bar{\theta}(\alpha), \alpha), 1]$, it is optimal to have a lower threshold $\bar{\theta}(\alpha)$ when α is large. Therefore, $q(\bar{\theta}(\alpha_1), \alpha_1) < q(\bar{\theta}(\alpha_1), \alpha_2)$ for $\alpha_1 > \alpha_2$, and so $q(\theta, \alpha)$ must be decreasing in α when θ is sufficiently high.

Essentially, when α is large, then $q(\cdot)$ is front-loaded: it is high when θ is small, and relatively low when α is large. The opposite is true when α is small.

Table 1 describes the aggregate welfare effects from the presence of honest types for the uniform-quadratic case of Corollary 1. The aggregate welfare gain WG is measured as a percentage of the maximal possible welfare gain relative to the standard second-best mechanism for an environment in which all consumers are strategic. In interpreting the table, one should keep in mind that the quantity schedules of honest and strategic consumers are both distorted relative to the first-best, but that the former is less distorted than the latter. When the fraction of honest consumers gets larger, welfare increases for two reasons: first, strategic consumers are replaced by honest ones; second, both quantity schedules become less distorted.

Finally, to elicit some empirically testable implications of our model, we examine the optimal tariff offered to the strategic consumers. The optimal menu $(q^s(\theta), t^s(\theta))$ from which strategic consumers choose can equivalently be interpreted as a tariff $T^s(q)$, where $q \in [0, q^s(1)]$. Specifically, $T^s(q) = t^s(\theta^s(q))$, where $\theta^s(\cdot)$ is the inverse of $q^s(\cdot)$. Maskin and Riley (1984) have shown that in the standard model where all consumers are strategic, the optimal tariff exhibits quantity discounts (i.e., $T^s(q)/q$ is decreasing) for all quantity (quality) levels, under constant marginal cost and plausible assumptions on the utility function and the distribution of types, including the case with linear-quadratic utility and uniform distribution of valuations. Maintaining the constant marginal cost assumption to make the comparison legitimate, we show that in the presence of honest consumers, the optimal tariff necessarily contains a region of quantity premia where $T^s(q)/q$ is increasing.

TABLE 1 **Welfare Gain from Honest Consumers**

% Honest	1	2	5	10	20	30	40	50	60	70	80	90	95
%WG	1.22	2.45	6.14	12.3	24.57	36.57	48.13	59.11	69.38	78.8	87.21	94.42	97.46

Note: %Honest $\equiv 100 \times \alpha/(1+\alpha)$, $\%WG(\alpha/(1+\alpha)) \equiv (W(\alpha) - W_{sb})/(W_{max} - W_{sb}) \times 100$, where $W_{max} \equiv \int_0^1 \theta q^*(\theta) - (q^*(\theta)^2/2) d\theta = 1/6$, $W_{sb} \equiv \int_0^1 \theta \max\{0, 2\theta - 1\} - [(\max\{0, 2\theta - 1\})^2/2] d\theta = 1/8$, and $W(\alpha) \equiv (\int_0^1 \theta q(\theta) - (q(\theta)^2/2) + \alpha(\theta g(\theta) - (g(\theta)^2/2)) d\theta)/(1+\alpha)$.

Theorem 5. Suppose that $c(q) = \tilde{c}q$ for some $\tilde{c} \geq 0$ and $u(\cdot)$ and $F(\cdot)$ satisfy Assumption 1. Then there exists $\widehat{q} > 0$ and $q^d < q^s(1)$ such that the optimal tariff $T^s(q)$ given to strategic consumers exhibits quantity premia on $(0, \widehat{q}]$ and quantity discounts on $(q^d, q^s(1)]$.

If $u(\theta, q) = \theta q - q^2/2$, $F(\cdot)$ is uniform, and $c(q) = \tilde{c}q$, then for every $\alpha > 0$ there exists $\widehat{q}_\alpha \in (0, 1)$ s.t. $T^s(q)$ exhibits quantity premia for $q < \widehat{q}_\alpha$ and quantity discounts for $q > \widehat{q}_\alpha$. The threshold $\widehat{q}_\alpha$ is increasing in α and converges to zero when α converges to zero.

In combination, the results of Maskin and Riley (1984) and Theorem 5 imply that it is optimal for the firm to charge quantity (quality) premia to strategic consumers at low quantity (quality) levels if and only if honest consumers are present. Note that, by Lemma A9 in the Appendix, at low quantity (quality) levels, honest and strategic consumers are charged the same amount for the same quantity. Thus, we obtain an empirically testable implication of our model. Specifically, to test for the presence of honest consumers, one can simply look for evidence of quantity premia in the data.

Moreover, it is possible to gauge the size of the fraction of honest consumers in the population by the range of the premia region: the larger this region is, the bigger the proportion of honest consumers in the population.

4. Conclusions

■ In this article we have offered an explanation for the frequently observed pricing practice of using information obtained directly from customers to sell the same quality or quantity to different consumers at different prices. Such pricing strategies would be infeasible if, as the traditional screening literature maintains, all consumers could easily and costlessly manipulate their private information.

We offered several explanations for why some consumers may not be able to misrepresent or conceal their preferences. Consumers may be boundedly rational and may not fully understand the implications of their responses for the terms of trade they will subsequently face. For ethical or moral reasons, consumers may be averse to lying. Finally, consumers may differ in their ability or cost of presenting evidence supporting their claims.

Econometric and experimental evidence suggests that the fraction of the population that act nonstrategically is nonnegligible, and may in fact be surprisingly large. We have shown that such behavior has important consequences for the optimal pricing policy of profit-maximizing firms. Specifically, the presence of honest consumers in the population reduces the allocational distortions associated with monopoly power. Furthermore, the traditional telltale sign of monopoly power, firms' refusal to serve customers whose value is close to the marginal cost of production, is simply absent in our model. Our model also offers an explanation why firms may wish to offer complicated or nontransparent mechanisms. Namely, when the seller faces some "boundedly rational" consumers, it is optimal for her to construct a mechanism in which finding discounts is difficult and requires analytical abilities that those customers lack.

Our theory has implications that go beyond the problem of optimal screening by profit-maximizing firms, or the closely related issue of optimal regulatory policy. Indeed, the traditional exclusion motive also appears forcefully in bilateral bargaining. With one-sided asymmetric information, exclusion manifests itself in the form of absence of intertemporal price discrimination or "haggling" (see Stokey, 1979; Riley and Zeckhauser, 1983). With two-sided asymmetric information, it appears in the form of no trade when the difference between seller's cost and buyer's valuation is small (see Myerson and Satterthwaite, 1983; Williams, 1987). The exclusion motive is also present in auctions where the seller selects an optimal reserve price above her true cost. Exploring the implications of our model for these environments is a topic we leave for future research.

One extension that is more immediate involves relaxing the assumption that a consumer's likelihood of telling the truth is independent of her valuation for the object. As a first approximation, this assumption is certainly reasonable. However, it would be comforting to know that our results are robust to perturbations in which the likelihood of being honest is allowed to depend upon the underlying valuation θ. Assuming that $\gamma(\theta)$ is smooth and satisfies $\gamma(\theta) > 0$ for all $\theta \in [0, 1]$, a careful perusal of our proofs reveals that results analogous to Theorems 3 and 4 hold. In particular, no consumer is ever excluded from the market. Furthermore, if the solution to the unconstrained problem (ignoring the monotonicity constraint on the quantity schedule for strategic types) is monotone, the optimal quantity schedule is again characterized by two differential equations, equation (12) and the following counterpart to equation (11):

$$\begin{aligned} q'\Bigg(&\max\left\{\gamma(r)f(r)\frac{u_q(q,r)-c'(q)}{u_\theta(q,r)},0\right\} \\ &+(1-\gamma(\theta))f(\theta)\left[\frac{(u_q(q,\theta)-c'(q))u_{\theta qq}(q,\theta)}{u_{\theta q}^2(q,\theta)}-\frac{u_{qq}(q,\theta)-c''(q)}{u_{\theta q}(q,\theta)}\right]\Bigg) \\ &=[(1-\gamma(\theta))f'(\theta)-f(\theta)\gamma'(\theta)]\frac{u_q(q,\theta)-c'(q)}{u_{\theta q}(q,\theta)} \\ &\quad+(1-\gamma(\theta))f(\theta)\left[2-\frac{(u_q(q,\theta)-c'(q))u_{\theta\theta q}(q,\theta)}{u_{\theta q}^2(q,\theta)}\right]. \end{aligned}$$

To understand this formula, note that the perturbed model is equivalent to the original one, but where there is now a density $(1-\gamma(\theta))f(\theta)$ of strategic consumers and a density $\gamma(\theta)f(\theta)$ of honest consumers.

A second extension allows there to be a finite number of types $0 = \theta_0 < \theta_1 < \cdots < \theta_n = \theta^u$. In this case, it is easy to show that if α is sufficiently small, then there can be exclusion on an initial segment $\{\theta_0, \ldots, \theta_{\hat{k}}\}$ of consumer valuations. Nevertheless, the following generalization of our no-exclusion result holds: for any $\alpha > 0$ as the type space becomes finer (i.e., the mesh of the partition of the interval $[0, \theta^u]$ converges to zero), the highest valuation in the set of excluded consumer types converges to zero, i.e., $\theta_{\hat{k}} \to 0$. So we recover our no-exclusion result in the limit as the type space becomes arbitrarily fine.

We expect that our no-exclusion result would also hold if the firm were initially uncertain about the fraction of honest consumers in the population but could learn more about its size over time. In this case, we can take $\gamma(\theta)$ to be the expected fraction of honest consumers with valuation θ. As observed above, as long as the firm's prior beliefs are such that $\gamma(\theta) > 0$ for all θ, no type would be excluded initially. Over time, as the firm continues to make sales to the consumers, it would learn more about the distribution of valuations in the population and update its beliefs accordingly. But Bayesian updating implies that $\gamma(\theta)$ would always remain above zero, so all consumers will be served at all times.

Finally, we believe that the no-exclusion result would also hold if consumers were risk averse with respect to money. Recall that the proof of the no-exclusion result is based on a perturbation argument: starting from an allocation profile in which all types with valuations below some $\underline{\theta} > 0$ are assigned zero quantities, we show that the firm can increase its expected profits by raising the quantities assigned to low-valuation types by some small amount. Since the required perturbation is small, we obtain a very accurate approximation by replacing each agent's utility function by its linearization in transfers—which takes us back to the quasi-linear model. Then, provided that the marginal utility of small transfers is not too sensitive to the agent's valuation, the argument used to prove no exclusion in the quasi-linear case would apply and all the order comparisons will remain valid for the linearized model, and hence for the true model as well—given the accuracy of the approximation. We leave the verification of the details of this argument for future research.

Appendix

■ The proofs of Theorems 1, 4, and 5 follow.

Proof of Theorem 1. Consider an allocation profile $\{q(\theta), g(\theta), t^s(\theta), t^\tau(\theta)\}$ implementable via some game form Γ. Let us show that it is also a.e. implementable via the password mechanism.

Since a strategic consumer can imitate any other type in any game form including Γ, this allocation profile must satisfy all incentive constraints of all strategic consumers (or, at least, a set of full measure of strategic consumers), i.e., the allocation $(q(\theta), t^s(\theta))$ has to provide more surplus to a consumer with valuation θ than any allocation from $(q(\tilde{\theta}), t^s(\tilde{\theta}))_{\tilde{\theta}\neq\theta}$, or any allocation from $(g(\tilde{\theta}), t^\tau(\tilde{\theta}))_{\tilde{\theta}\in[0,1]}$. So, a strategic consumer will not imitate another type if the principal offers $(q(\theta), g(\theta), t^s(\theta), t^\tau(\theta))$ in the password mechanism.

Further, in stage 1 of the password mechanism the only feasible reporting strategy for an honest consumer with valuation $\theta > 0$ is to report her true valuation. So, an honest consumer with valuation θ cannot deviate from the allocation $(g(\theta), t^\tau(\theta))$.

Thus, at most one consumer type—an honest consumer with valuation zero who gets access to the menu $\{q(\theta), t^s(\theta) \mid \theta \in [0, 1]\}$—may choose to deviate from an allocation designed for her. So an allocation profile satisfying all incentive constraints of the strategic consumers is a.e. implementable via the password mechanism.[30] *Q.E.D.*

Proof of Theorem 4. We start by establishing several properties of a solution.

Lemma A1. If $q(\theta)$ is an optimal quantity schedule, then it is continuous.

Lemma A2. Let $U_\tau(\theta) = u(g(\theta), \theta) - t^\tau(\theta)$, and suppose that $U_\tau(\theta_2) - U_\tau(\theta_1) > u(g, \theta_2) - u(g, \theta_1)$ for some θ_1, θ_2 s.t. $\theta_2 > \theta_1$ and g. Then $U_\tau(\theta_4) - U_\tau(\theta_3) > u(g, \theta_4) - u(g, \theta_3)\ \forall\theta_3, \theta_4$ s.t. $\theta_4 > \theta_3 \geq \theta_2$.

Lemma A3. In an optimal mechanism, $U(0) = 0$.

Lemma A4 (common cutoff). The optimal quantity schedules are such that $q(\theta) = 0$ if and only if $g(\theta) = 0$.

Proof. Suppose that there exists θ s.t. $q(\theta) = 0$ and $g(\theta) > 0$. By continuity of $q(\theta)$, $\exists\theta' > \theta$ s.t. $q(\theta') < g(\theta)$. Since $q(\theta)$ is nondecreasing, $U(\theta') = \int_0^{\theta'} u_\theta(q(s), s)ds < u(g(\theta), \theta') - u(g(\theta), \theta)$ i.e., $ICT(\theta', \theta)$ in (7) fails.

Next, suppose that $\exists\tilde{\theta} > 0$ s.t. $g(\tilde{\theta}) = 0$ and $q(\tilde{\theta}) > 0$. By Lemma 1, $\tilde{\theta} > 0$. Since $g(\cdot)$ is nondecreasing and $q(\cdot)$ is continuous, $\exists\varepsilon > 0$ s.t. $q(\theta) > 0 = g(\theta)\ \forall\,\theta \in [\tilde{\theta} - \varepsilon, \tilde{\theta}]$.

But then without violating any incentive constraints in (7), the firm can increase its profits by setting $g(\theta) = \min\{q^*(\theta), q(\theta)\} > 0$ where $q^*(\theta)$ is the first-best quantity for 0 and $t^\tau(\theta) = u(g(\theta), \theta)\ \forall\,\theta \in [\tilde{\theta} - \varepsilon, \tilde{\theta}]$. *Q.E.D.*

Let $q^{sb}(\theta)$ be an optimal quantity schedule in the standard case without honest consumers, i.e., when $\alpha = 0$.[31] The following lemma compares an optimal quantity schedule $q(\cdot)$ to the benchmarks $q^*(\theta)$ and $q^{sb}(\theta)$.

Lemma A5. For any $\alpha > 0$, an optimal quantity schedule $q(\cdot)$ satisfies $q^{sb}(\theta) \leq q(\theta) \leq q^*(\theta)\ \forall\,\theta \in (0, 1]$.

The next lemma is a key step in solving our two-dimensional screening problem. It shows that the family of incentive constraints $ICT(\theta, \theta')$ in (7) can be replaced with a simpler one.

Lemma A6. Assume $U(0) = 0$. For any nondecreasing continuous function $q(\theta)$, define $r(\theta)$ as follows. When θ is such that $q(\theta) > 0$, let $r(\theta)$ be the unique solution to $U(\theta) = u(q(\theta), \theta) - u(q(\theta), r(\theta))$. When θ is such that $q(\theta) = 0$, let $r(\theta) = \theta$. Then:

(i) $r(\theta)$ is a nondecreasing function satisfying $r(0) = 0$. Furthermore, it is strictly increasing (continuous) at all θ, where $q(\cdot)$ is strictly increasing (continuous).

(ii) Suppose that $g(\theta) \leq q(1) = q^*(1)$. Then $U(\theta) \geq u(g(\theta'), \theta) - u(g(\theta'), \theta')\ \forall\,\theta, \theta' \in [0, 1]$ if and only if $q(\theta) \geq g(r(\theta))\ \forall\,\theta \in [0, 1]$.

Proof. (i) Since $q(\theta)$ is nondecreasing and $U(0) = 0$, we have $0 \leq U(\theta) = \int_0^\theta u_\theta(q(s), s)ds \leq u(q(\theta), \theta)$. It is then immediate from the definition of $r(\theta)$ that $0 \leq r(\theta) \leq \theta$ and so $r(0) = 0$. Let $\theta_2 > \theta_1$ and $q(\theta_1) > 0$. Then, using the definition of $r(\theta)$ and the fact that $q(\cdot)$ is nondecreasing, we have

$$\begin{aligned} u(q(\theta_2), r(\theta_2)) - u(q(\theta_1), r(\theta_1)) &= u(q(\theta_2), \theta_2) - u(q(\theta_1), \theta_1) - \int_{\theta_1}^{\theta_2} u_\theta(q(s), s)ds \\ &\geq u(q(\theta_2), \theta_1) - u(q(\theta_1), \theta_1). \end{aligned} \tag{A1}$$

The first equality implies that $r(\theta_2) = r(\theta_1)$ if $q(\theta_2) = q(\theta_1)$, while the inequality in combination with the fact that $u_{\theta q}(q, \theta) > 0$ implies that $r(\theta_2) > r(\theta_1)$ if $q(\theta_2) > q(\theta_1)$.

Next, define $\underline{\theta} \equiv \sup\{\theta' \mid q(\theta') = 0\}$. Since $U(\theta), q(\theta)$ and $u(q, s)$ are continuous, it follows that $r(\theta)$ is continuous at $\theta > \underline{\theta}$. Since $q(\cdot)$ is nondecreasing, $q(\theta) = 0$ and hence by definition $r(\theta) = \theta$ for all $\theta < \underline{\theta}$. Thus $r(\theta)$ is continuous on $[0, \underline{\theta})$. To establish the continuity of $r(\theta)$ at $\underline{\theta}$, note that $\forall\,\theta > \underline{\theta}$ we have

$$\int_{r(\theta)}^{\theta} u_\theta(q(\theta), s)ds = u(q(\theta), \theta) - u(q(\theta), r(\theta)) = \int_{\underline{\theta}}^{\theta} u_\theta(q(s), s)ds \leq \int_{\underline{\theta}}^{\theta} u_\theta(q(\theta), s)ds.$$

[30] We show below that the optimal allocation is such that the honest consumer with valuation zero does not wish to deviate by choosing an allocation from $\{q(\tilde{\theta}), t^s(\tilde{\theta}) \mid \tilde{\theta} \in [0, 1]\}$ instead of $(g(0), t^\tau(0))$. So, in fact, the optimal allocation profile is implemented for all types. It is formally correct to consider a.e. implementation here, because no incentive constraints are imposed on $(g(0), t^\tau(0))$.

[31] Under the assumptions of Theorem 4, $q^{sb}(\theta) = \arg\max_q\{(u(q, \theta) - c(q))f(\theta) - u_\theta(q, \theta)(1 - F(\theta))\}$.

The last inequality can hold only if $r(\theta) \geq \underline{\theta}$. Since $r(\theta) \leq \theta$ for all θ, we conclude that the right-hand limit of $r(\theta)$ at $\underline{\theta}$ is equal to $\underline{\theta}$.

(ii) Suppose that $ICT(\theta, \theta')$ in (7) holds $\forall\, \theta, \theta' \in [0, 1]$. If $q(\theta) = 0$, then $r(\theta) = \theta$. By Lemma A4, we have $g(\theta) = 0$, so $g(r(\theta)) = g(\theta) = q(\theta) = 0$.

If $q(\theta) > 0$, then using the definition of $r(\theta)$ and the fact that $ICT(\theta, r(\theta))$ holds, we have

$$u(q(\theta), \theta) - u(q(\theta), r(\theta)) = U(\theta) \geq u(g(r(\theta)), \theta) - u(g(r(\theta)), r(\theta)).$$

Then the single-crossing implies that $q(\theta) \geq g(r(\theta))$.

Now suppose that $q(\theta) \geq g(r(\theta))\, \forall\, \theta \in [0, 1]$. Consider any pair $(\theta, \widehat{\theta})$. Since $r(\theta)$ is continuous and nondecreasing and $r(0) = 0$, either $\widehat{\theta} = r(\tilde{\theta})$ for some $\tilde{\theta} \in [0, 1]$ or $\widehat{\theta} > r(1)$. If $\widehat{\theta} = r(\tilde{\theta})$ for some $\tilde{\theta} \in [0, 1]$,[32] then by assumption $g(\widehat{\theta}) \leq q(\tilde{\theta})$. Take any $\theta \geq \tilde{\theta}$. Then

$$U(\theta) = \int_{\tilde{\theta}}^{\theta} u_\theta(q(s), s)ds + U(\tilde{\theta}) \geq u(q(\tilde{\theta}), \theta) - u(q(\tilde{\theta}), \tilde{\theta}) + u(q(\tilde{\theta}), \tilde{\theta}) - u(q(\tilde{\theta}), \widehat{\theta}) \geq u(g(\widehat{\theta}), \theta) - u(g(\widehat{\theta}), \widehat{\theta}). \quad \text{(A2)}$$

The first inequality follows from the fact that $q(\cdot)$ is nondecreasing and the definition of $r(\tilde{\theta})$, while the second inequality follows from single-crossing and $q(\tilde{\theta}) \geq g(\widehat{\theta})$. So, $ICT(\theta, \widehat{\theta})$ holds.

If $\widehat{\theta} < \theta < \tilde{\theta}$, then $q(\theta) \leq q(\tilde{\theta})$. So, $ICT(\theta, \widehat{\theta})$ holds because

$$U(\theta) = U(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} u_\theta(q(s), s)ds \geq u(q(\tilde{\theta}), \tilde{\theta}) - u(q(\tilde{\theta}), \widehat{\theta}) - \left(u(q(\tilde{\theta}), \tilde{\theta}) - u(q(\tilde{\theta}), \theta)\right) \geq u(g(\widehat{\theta}), \theta) - u(g(\widehat{\theta}), \widehat{\theta}).$$

Finally, suppose that $\widehat{\theta} > r(1)$. Since $g(\widehat{\theta}) \leq g^*(1) = q(1)$, for $\theta > \widehat{\theta}$ we have

$$\begin{aligned} U(\theta) = U(1) - \int_{\theta}^{1} u_\theta(q(s), s)ds &\geq u(q(1), 1) - u(q(1), r(1)) - (u(q(1), 1) - u(q(1), \theta)) \\ &= u(q(1), \theta) - u(q(1), r(1)) \geq u(g(\widehat{\theta}), \theta) - u(g(\widehat{\theta}), r(1)) > u(g(\widehat{\theta}), \theta) - u(g(\widehat{\theta}), \widehat{\theta}). \end{aligned}$$

Thus, $ICT(\theta, \widehat{\theta})$ also holds in this case. *Q.E.D.*

Since $r(\theta)$ is continuous and nondecreasing and $r(0) = 0$, $r(\cdot)$ maps $[0, 1]$ onto $[0, r(1)]$. The inverse image $r^{-1}(\widehat{\theta})$ from $[0, r(1)]$ is unique if $q(\cdot)$ is strictly increasing at θ s.t. $r(\theta) = \widehat{\theta}$. However, even if $r^{-1}(\widehat{\theta})$ is not unique, $q(r^{-1}(\widehat{\theta}))$ is unique, because $r(\theta_1) = r(\theta_2)$ only if $q(\theta_1) = q(\theta_2)$. Lemma A6 allows us to establish the following important result.

Lemma A7. Fix a nondecreasing continuous quantity schedule $q(\theta)$ s.t. $q(1) = q^*(1)$, and set $U(0) = 0$. Then the optimal quantity schedule $g(\theta)$ that maximizes (6) subject to (7) is given by

$$g(\theta) = \begin{cases} \min\{q^*(\theta), q(r^{-1}(\theta))\} & \text{if } \theta \leq r(1) \\ q^*(\theta) & \text{if } \theta > r(1). \end{cases}$$

Proof. By Lemma 1, $g(\theta) \leq q^*(1)$. So, by (ii) of Lemma A6, the family $ICT(\theta, \theta')$ of incentive constraints in (7) can be replaced with the following family: $q(\theta) \geq g(r(\theta))\, \forall\, \theta \in [0, 1]$, which can be rewritten as $q(r^{-1}(\theta)) \geq g(\theta)$ $\forall\, \theta \in [0, r(1)]$. Then the result follows because the integrand $u(g(\theta), \theta) - c(g(\theta))$ of the second integral in (6) is strictly concave in $g(\theta)$ and is strictly increasing (decreasing) in $g(\theta)$ if $g(\theta) < q^*(\theta)$ $(g(\theta) > q^*(\theta))$. *Q.E.D.*

Lemma A7 implies that $g(\cdot)$ is completely determined by $q(\cdot)$. So $q(\cdot)$ remains the only choice variable, which reduces the dimensionality of our problem. By Lemmas 1 and A1–A3, we can without loss of generality impose the following additional restrictions on the domain in problem (6): $q(\cdot)$ is continuous, $q(0) = 0$, $g(\theta) \leq q(1) = q^*(1)$, $U(0) = 0$. Therefore, problem (6) is equivalent to the following one:

$$\begin{aligned} \max_{q(\theta)\geq 0} \int_0^1 \left(u(q(\theta), \theta) - c(q(\theta)) - u_\theta(q(\theta), \theta)\frac{1 - F(\theta)}{f(\theta)}\right) f(\theta)d\theta + \alpha \int_{r(1)}^{1} (u(q^*(\tilde{\theta}), \tilde{\theta}) - c(q^*(\tilde{\theta})))f(\tilde{\theta})d\tilde{\theta} \\ + \alpha \int_0^{r(1)} \left(u(\min\{q^*(\tilde{\theta}), q(r^{-1}(\tilde{\theta}))\}, \tilde{\theta}) - c(\min\{q^*(\tilde{\theta}), q(r^{-1}(\tilde{\theta}))\})\right) f(\tilde{\theta})d\tilde{\theta} \qquad \text{(A3)} \end{aligned}$$

subject to: $q(\cdot)$ is nondecreasing, continuous, $q(0) = 0$, and $q(1) = 1$.

This characterization of the monopolist's maximization problem will be used to prove Theorem 3, our no-exclusion result.

Our next step is to reformulate problem (A3) to make it amenable to standard methods of optimal control. We will need to use the derivative $q'(\cdot)$ as the control variable, and so $q'(\cdot)$ has to be piecewise continuous or, equivalently,

[32] $\tilde{\theta}$ need not be unique. However, if $\widehat{\theta} = r(\tilde{\theta}_1) = r(\tilde{\theta}_2)$, then as established above $q(\tilde{\theta}_1) = q(\tilde{\theta}_2)$.

$q(\cdot)$ has to belong to the space $C_p^1([0,1])$ of piecewise smooth (continuous and piecewise continuously differentiable) functions on $[0, 1]$. So far, we have only established that $q(\cdot)$ must be continuous, i.e., $q(\cdot) \in C([0,1])$, and that $q'(\cdot)$ exists almost everywhere. Nevertheless, the next lemma demonstrates that we can without loss of generality assume that $q(\cdot) \in C_p^1([0,1])$.

Lemma A8. If $q(\theta)$ is a solution to the maximization problem (A3) on the domain $C_p^1([0,1])$, then $q(\theta)$ also maximizes (A3) on the domain $C([0,1])$.

Next, let us make a change of variables $\tilde{\theta} = r(\theta)$ in the third term of (A3). By Lemma A6, $r(\theta)$ is continuous, increasing, and bounded on $[0, 1]$. Therefore, it is Riemann integrable and the change of variables is legitimate. Further, Theorem 3 implies that $U(\theta) = u(q(\theta),\theta) - u(q(\theta), r(\theta))\ \forall\ \theta \in (0, 1]$. Since $q(\theta) \in C_p^1([0,1])$, we can differentiate this expression at all but (at most) a finite set of θ's to yield

$$r'(\theta) = \frac{q'(\theta)(u_q(q(\theta),\theta) - u_q(q(\theta), r(\theta)))}{u_\theta(q(\theta), r(\theta))}. \tag{A4}$$

Note that $r'(\theta)$ is piecewise continuous since $q'(\theta)$ is piecewise continuous. So, the change of variables $\tilde{\theta} = r(\theta)$ allows us to express the third term in (A3) as the following Riemann integral:

$$\alpha \int_0^1 \left(u(\min\{q^*(r(\theta)), q(\theta)\}, r(\theta)) - c(\min\{q^*(r(\theta)), q(\theta)\})\right) f(r(\theta))r'(\theta)d\theta.$$

Using (A4), we can finally restate problem (A3) as follows:

$$\max_{q(\cdot)} \int_0^1 \left(u(q(\theta),\theta) - c(q(\theta)) - u_\theta(q(\theta),\theta)\frac{1-F(\theta)}{f(\theta)}\right) f(\theta)d\theta + \alpha \int_{r(1)}^1 (u(q^*(\theta),\theta) - c(q^*(\theta)))f(\theta)d\theta$$
$$+ \alpha \int_0^1 \left(u(\min\{q^*(r(\theta)), q(\theta)\}, r(\theta)) - c(\min\{q^*(r(\theta)), q(\theta)\})\right) f(r(\theta))\frac{q'(\theta)(u_q(q(\theta),\theta) - u_q(q(\theta), r(\theta)))}{u_\theta(q(\theta), r(\theta))}d\theta \tag{A5}$$

subject to

$$r'(\theta) = \frac{q'(\theta)(u_q(q(\theta),\theta) - u_q(q(\theta), r(\theta)))}{u_\theta(q(\theta), r(\theta))}, \quad q(0) = 0, \quad r(0) = 0, \quad q(1) = q^*(1), \quad \text{and} \quad q'(\theta) \geq 0. \tag{A6}$$

Observe that (A5) and (A6) is an optimal control problem with control variable $q'(\theta)$, two state variables $q(\theta)$ and $r(\theta)$, and a restriction on the control $q'(\cdot) \geq 0$. It has "scrap value" $S(r(1)) = \alpha \int_{r(1)}^1 (u(q^*(\theta),\theta) - c(q^*(\theta)))f(\theta)d\theta$ at $\theta = 1$.

The existence of a solution to this problem follows from the Filippov-Cesari theorem (see Seierstad and Sydsæter, 1987). Pontryagin's Maximum Principle provides a standard method of solution to this problem. The corresponding Hamiltonian is given by

$$\begin{aligned} &H(q, r, \lambda, \delta, \theta) \\ &\quad = \left(u(q,\theta) - c(q) - u_\theta(q,\theta)\frac{1-F(\theta)}{f(\theta)}\right) f(\theta) \\ &\qquad + \alpha f(r)\left(u(\min\{q^*(r), q\}, r) - c(\min\{q^*(r), q\})\right) \frac{q'(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)} + \lambda q' + \delta\frac{q'(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)}, \end{aligned} \tag{A7}$$

where $\lambda(\theta)$ and $\delta(\theta) \in C_p^1([0,1])$ are costate variables associated with the laws of motion of $q(\theta)$ and $r(\theta)$, respectively. Incorporating the constraint $q'(\theta) \geq 0$, we obtain the following Lagrangian:

$$\mathcal{L} = H(q, r, \lambda, \delta, \theta) + \tau q', \tag{A8}$$

where $\tau \geq 0$ and $\tau q' = 0$. The transversality condition on the costate variable δ is

$$\delta(1) = dS(r_1)/dr_1 = -\alpha(u(q^*(r(1)), \quad r(1)) - c(q^*(r(1))))f(r(1)).$$

According to the Maximum Principle, the necessary conditions for an optimum are $-\lambda'(\theta) = \partial H/\partial q$, $-\delta'(\theta) = \partial H/\partial r$, and $\partial\mathcal{L}/\partial q' = 0$. The form of these conditions depends on the sign of $q(\theta) - q^*(r(\theta))$. Consequently, we need to consider two cases.

Case 1. $q < q^*(r)$. The necessary first-order conditions are

$$\begin{aligned} -\lambda'(\theta) = \frac{\partial H}{\partial q} &= \left(u_q(q,\theta) - c'(q) - u_{\theta q}(q,\theta)\frac{1-F(\theta)}{f(\theta)}\right) f(\theta) + \alpha f(r)(u_q(q,r) - c'(q))\frac{q'(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)} \\ &\quad + (\alpha f(r)(u(q,r) - c(q)) + \delta)\, q' \left(\frac{u_{qq}(q,\theta) - u_{qq}(q,r)}{u_\theta(q,r)} - \frac{(u_q(q,\theta) - u_q(q,r))u_{\theta q}(q,r)}{u_\theta(q,r)^2}\right), \end{aligned} \tag{A9}$$

$$-\delta'(\theta) = \frac{\partial H}{\partial r} = \left(\alpha f'(r)(u(q,r) - c(q)) + \alpha f(r) u_\theta(q,r)\right) \frac{q'(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)}$$
$$- (\alpha f(r)(u(q,r) - c(q)) + \delta)\, q' \left(\frac{u_{q\theta}(q,r)}{u_\theta(q,r)} + \frac{(u_q(q,\theta) - u_q(q,r)) u_{\theta\theta}(q,r)}{u_\theta(q,r)^2}\right), \quad \text{(A10)}$$
$$0 = \frac{\partial \mathcal{L}}{\partial q'} = (\alpha f(r)(u(q,r) - c(q)) + \delta) \frac{(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)} + \lambda + \tau. \quad \text{(A11)}$$

To solve (A9)–(A11), first use (A9) and (A10) to solve for $-[d(\lambda + \delta(u_q(q,\theta) - u_q(q,r))/u_\theta(q,r))]/d\theta$. Second, differentiate (A11) to obtain another expression for $-[d(\lambda + \delta(u_q(q,\theta) - u_q(q,r))/u_\theta(q,r))]/d\theta$. Equating the two expressions yields

$$[\delta(\theta) + \alpha f(r)(u(q,r) - c(q))] \frac{u_{\theta q}(q,\theta)}{u_\theta(q,r)} + \tau' = (u_q(q,\theta) - c'(q)) f(\theta) - u_{\theta q}(q,\theta)(1 - F(\theta)). \quad \text{(A12)}$$

Consider any interval (θ_1, θ_2) where $q'(\theta) > 0$. In that case, $\tau(\theta) = 0$ and so $\tau'(\theta) = 0$. Then by (A12),

$$\delta(\theta) = \frac{(u_q(q,\theta) - c'(q)) u_\theta(q,r)}{u_{\theta q}(q,\theta)} f(\theta) - u_\theta(q,r)(1 - F(\theta)) - \alpha f(r)(u(q,r) - c(q)). \quad \text{(A13)}$$

Differentiate this expression to get the first expression for $\delta'(\theta)$. The second expression for $\delta'(\theta)$ is obtained by using (A12) to substitute δ out from the right-hand side of (A11). Equating the two expressions for $\delta'(\theta)$, we get the following differential equation:

$$q' \left(\alpha \frac{f(r)(u_q(q,r) - c'(q))}{u_\theta(q,r)} + f(\theta) \frac{(u_q(q,\theta) - c'(q)) u_{\theta qq}(q,\theta)}{u_{\theta q}(q,\theta)^2} - f(\theta) \frac{u_{qq}(q,\theta) - c''(q)}{u_{\theta q}(q,\theta)}\right)$$
$$= 2f(\theta) + f'(\theta) \frac{u_q(q,\theta) - c'(q)}{u_{\theta q}(q,\theta)} - f(\theta) \frac{(u_q(q,\theta) - c'(q)) u_{\theta\theta q}(q,\theta)}{u_{\theta q}(q,\theta)^2}. \quad \text{(A14)}$$

So when $q' > 0$, the solution in case 1 is characterized by (A14) and the "law of motion" (A4).

Next, suppose there is an interval $[\theta_1, \theta_2]$ where the monotonicity constraint $q' = 0$ is binding. Such an interval will occur in case 1 if the solution to (A14) is nonmonotone. To derive the correct $q(\cdot)$, we need to apply a modified version of the so-called ironing technique (see Guesnerie and Laffont, 1984). Specifically, if $q' = 0$, then $r' = 0$, so (A10) implies that $\delta' = 0$. If θ_1 and θ_2 are endpoints of an interval on which $q(\cdot)$ is constant, i.e., $q(\theta)$ is strictly increasing on $(\theta_1 - \varepsilon_1, \theta_1)$ and $(\theta_2, \theta_2 + \varepsilon_2)$ for some $\varepsilon_1, \varepsilon_2 > 0$, then by continuity we have $\tau(\theta_2) = \tau(\theta_1) = 0$. Integrating (A12) on $[\theta_1, \theta_2]$ yields

$$(\alpha f(r)(u(q,r) - c(q)) + \delta) \frac{u_q(q,\theta_1) - u_q(q,\theta_2)}{u_\theta(q,r)} = (u_q(q,\theta_2) - c'(q))(1 - F(\theta_2) - (u_q(q,\theta_1) - c'(q))(1 - F(\theta_1)). \quad \text{(A15)}$$

Equation (A15) characterizes the interval(s) on which q is constant. Note that $\delta(\cdot)$ is constant on any interval where $q'(\cdot) = 0$, and $\delta(\theta_1)$ in case 1 is given by (A13). In the standard case without honest consumers, the counterpart of equation (A15) has the same right-hand side but has zero on the left-hand side. The remaining details of the ironing algorithm of Guesnerie and Laffont (1984) also apply in our case, and we refer the interested reader to their work.

Case 2. $q \geq q^*(r)$. The necessary first-order conditions are

$$-\lambda'(\theta) = \frac{\partial H}{\partial q} = \left(u_q(q,\theta) - c'(q) - u_{\theta q}(q,\theta) \frac{1 - F(\theta)}{f(\theta)}\right) f(\theta)$$
$$+ \left(\alpha f(r)(u(q^*(r),r) - c(q^*(r))) + \delta\right) q' \left(\frac{u_{qq}(q,\theta) - u_{qq}(q,r)}{u_\theta(q,r)} - \frac{(u_q(q,\theta) - u_q(q,r)) u_{\theta q}(q,r)}{u_\theta(q,r)^2}\right), \quad \text{(A16)}$$
$$-\delta'(\theta) = \frac{\partial H}{\partial r} = \left(\alpha f'(r)(u(q^*(r),r) - c(q^*(r))) + \alpha f(r) u_\theta(q^*(r),r)\right) \frac{q'(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)}$$
$$- \left(\alpha f(r)(u(q^*(r),r) - c(q^*(r))) + \delta\right) q' \left(\frac{u_{q\theta}(q,r)}{u_\theta(q,r)} + \frac{(u_q(q,\theta) - u_q(q,r)) u_{\theta\theta}(q,r)}{u_\theta(q,r)^2}\right), \quad \text{(A17)}$$
$$0 = \frac{\partial \mathcal{L}}{\partial q'} = \left(\alpha f(r)(u(q^*(r),r) - c(q^*(r))) + \delta\right) \frac{(u_q(q,\theta) - u_q(q,r))}{u_\theta(q,r)} + \lambda + \tau. \quad \text{(A18)}$$

Rearranging (A18), we obtain

$$\frac{d\left(\delta + f(r)(u(q^*(r),r) - c(q^*(r)))\right)}{d\theta} u_\theta(q,r) = \left(\delta + f(r)(u(q^*(r),r) - c(q^*(r)))\right) \frac{du_\theta(q,r)}{d\theta}.$$

Hence,

$$\delta + f(r)(u(q^*(r),r) - c(q^*(r))) = u_\theta(q,r) k_2, \quad \text{(A19)}$$

where k_2 is a constant of integration. Substituting this expression into (A18), we obtain that $-(\lambda + \tau) = k_2(u_q(q,\theta) - u_q(q,r))$. Differentiate this expression, substitute the result into (A16), and use (A19) on the right-hand side of (A16) to obtain

$$\tau'(\theta) + k_2 u_{\theta q}(q,\theta) = (u_q(q,\theta) - c'(q))f(\theta) - u_{\theta q}(q,\theta)(1 - F(\theta)). \tag{A20}$$

Equations (A19) and (A20) imply

$$[\delta + f(r)(u(q^*(r),r) - c(q^*(r)))]\frac{u_{\theta q}(q,\theta)}{u_\theta(q,r)} + \tau'(\theta) = (u_q(q,\theta) - c'(q))f(\theta) - u_{\theta q}(q,\theta)(1 - F(\theta)), \tag{A21}$$

which is identical to (A12).

On any interval where $q' > 0$ we have $\tau = \tau' = 0$, and so the solution is characterized by

$$(u_q(q,\theta) - c'(q))f(\theta) = u_{\theta q}(q,\theta)(1 + k_2 - F(\theta)). \tag{A22}$$

Lemma A5 implies that the constant of integration k_2 is nonpositive. Totally differentiating (A22), we obtain an equation that is identical to (A14) but without the term $\alpha[f(r)(u_q(q,r) - c'(q))]/u_\theta(q,r)$ on the left-hand side. In case 2, $u_q(q,r) - c'(q) \leq 0$, so (11) follows.

If the solution to (A22) is not monotonically increasing in θ, then we need to use an ironing procedure and determine the intervals on which $q(\cdot)$ is constant. To do so, note that if $q(\cdot)$ is constant on the interval $[\theta_1,\theta_2]$ and is strictly increasing immediately outside it, then $\tau(\theta_1) = \tau(\theta_2) = 0$ and $r'(\theta) = \delta'(\theta) = 0$ for all $\theta \in [\theta_1,\theta_2]$. Integrating (A21) on $[\theta_1,\theta_2]$ we obtain (A15), where δ can be computed using (A19). Finally, note that the solution cannot switch from one case to another on any interval where $q(\cdot)$ is constant, because $r(\cdot)$ is also constant on such interval.

Lemmas A9–A11 below help us to understand the overall properties of the solution.

Lemma A9. There exist θ_ℓ and θ_h satisfying $0 < \theta_\ell < \theta_h < 1$ such that case 1 applies on $(0,\theta_\ell)$, and case 2 applies and $q(\theta) = q^{sb}(\theta)$ on $(\theta_h, 1]$.

Lemma A9 leaves open the possibility that in the intermediate region $[\theta_\ell,\theta_h]$ the solution may switch between cases 1 and 2. Under some additional regularity conditions, however, this will not happen. More precisely, suppose that the hypothesis of Theorem 4 is satisfied, so that the optimal $q(\cdot)$ does not need ironing. Our next lemma then asserts that $\theta_\ell = \theta_h$, provided $u_{qq\theta}(q,\theta)$ is not too large and marginal social surplus is concave. Define

$$M = \max_{\theta\in[0,1], q\in[0,q^*(1)]} \frac{u_{q\theta}(q,\theta)}{u_\theta(q,\theta)}$$

$$N = \max_{\theta\in[0,1], q\in[0,q^*(1)]} \left(\frac{u_{\theta\theta}(q,\theta)}{u_\theta(q,\theta)} - \frac{u_{q\theta\theta}(q,\theta)}{u_{q\theta}(q,\theta)}\right)\left(\frac{c''(q) - u_{qq}(q,\theta)}{u_{q\theta}(q,\theta)}\right).$$

Lemma A10. Suppose that $f(\theta)[(u_q(q,\theta) - c'(q))/u_{\theta q}(\theta,q)]$ is strictly increasing in θ. Suppose furthermore that $\max_{\theta\in[0,1], q\in[0,q^*(1)]}[u_{qq\theta}(q,\theta)/u_{q\theta}(q,\theta)] \leq \min\{M,N\}$ and that $u_{qqq}(q,\theta) - c'''(q) \leq 0$. Then $\theta_\ell = \theta_h \equiv \overline{\theta}$, i.e., there exists a unique switchpoint between cases 1 and 2.

The system consisting of (11)–(12) with the boundary conditions $q(0) = r(0) = 0, q(1) = q^*(1)$ may have multiple solutions, i.e., there may exist several pairs $(q(\cdot), r(\cdot))$ satisfying the necessary conditions for optimality. Since our problem does not satisfy the standard sufficiency conditions (Arrow's or Mangasarian's), the optimal pair will have to be chosen by comparing the values of the objective. Our final lemma provides a sufficient condition under which this complication does not arise.

Lemma A11. Suppose that $(f(\theta)(u_q(q,\theta) - c'(q)))/u_\theta(q,\theta)$ is increasing in θ for all $\theta \in (0,1]$ and $q \in (q^{sb}(\theta), q^*(\theta)]$. Then there exists a unique solution to (11) and (12) satisfying the boundary conditions $q(0) = r(0) = 0$ and $q(1) = 1$.

Lemmas A9–A11 have an important implication. Suppose the conditions of Lemmas A10 and A11 hold. Then, to identify the solution to problem (A5) it is sufficient to find the solution $(\tilde{q}(\theta), \tilde{r}(\theta))$ to the system (12) and (A14) such that $q(0) = r(0) = 0$ and $\tilde{q}(\overline{\theta}) = q^*(\tilde{r}(\overline{\theta})) = q^{sb}(\overline{\theta})$ for some $\overline{\theta} \in (0,1)$. In combination, Theorem 4 and Lemma A9 imply that $\tilde{q}(\theta) < q^*(\tilde{r}(\theta))\ \forall\, \theta \in (0,\overline{\theta})$. So the optimal quantity schedule $q(\theta)$ is given by $q(\theta) = \tilde{q}(\theta)$ (case 1) for $\theta \in [0,\overline{\theta}]$ and $q(\theta) = q^{sb}(\theta)$ (case 2) for $\theta \in [\overline{\theta}, 1]$.

Proof of Theorem 3. Suppose that $\underline{\theta} \equiv \inf\{\theta | q(\theta > 0\} > 0$. We will show that there exists $\varepsilon > 0$, s.t. the firm can strictly increase its profits by (i) replacing the quantity schedule $q(\theta)$ with $\tilde{q}(\theta) = \max\{\varepsilon, q(\theta)\}$ for $\theta \in [\underline{\theta}/2, 1]$ (which is accomplished by adding the pair $(\varepsilon, u(\underline{\theta}/2, \varepsilon))$ to the menu offered to the strategic consumers) and (ii) modifying the quantity allocations to the honest consumerss so that the incentive compatibility is not violated.

Since $q(\theta)$ is continuous and nondecreasing, for any sufficiently small $\varepsilon > 0$ there exists a unique $\widehat{\theta}(\varepsilon)$ nondecreasing in ε s.t. $q(\widehat{\theta}(\varepsilon)) = \varepsilon$ and $q(\theta) > \varepsilon\ \forall\, \theta \in (\widehat{\theta}(\varepsilon), 1]$.[33] Under the modified schedule $\tilde{q}(\theta)$, strategic consumer obtains surplus $\tilde{U}(\theta) = u(\varepsilon,\theta) - u(\varepsilon,\underline{\theta}/2)\ \forall\, \theta \in [\underline{\theta}/2, \widehat{\theta}(\varepsilon)]$, and $\tilde{U}(\theta) = u(\varepsilon,\widehat{\theta}(\varepsilon)) - u(\varepsilon,\underline{\theta}/2) + \int_{\widehat{\theta}(\varepsilon)}^{\theta} u_\theta(q(s),s)ds\ \forall\, \theta \in [\widehat{\theta}(\varepsilon), 1]$.

[33] If such $\widehat{\theta}(\varepsilon)$ fails to exist for all $\varepsilon > 0$, then $q(\theta) = 0$ everywhere, which is suboptimal.

Define $\tilde{r}(\theta)$ as the solution to $\tilde{U}(\theta) = u(\tilde{q}(\theta), \theta) - u(\tilde{q}(\theta), \tilde{r}(\theta))$. By Lemma A7, $g(\theta) = \min\{q^*(\theta), q(r^{-1}(\theta))\}$, and $\tilde{g}(\theta) = \min\{q^*(\theta), \tilde{q}(\tilde{r}^{-1}(\theta))\}$. Also, define $G(q, \theta) = u(q, \theta) - c(q) - u_\theta(q, \theta)[(1 - F(\theta))/f(\theta)]$. According to (6), after this modification the firm's expected profit changes by

$$\Delta(\varepsilon) = \int_0^1 (G(\tilde{q}(\theta), \theta) - G(q(\theta), \theta))\, f(\theta)d\theta + \alpha \int_0^1 (u(\tilde{g}(\theta), \theta) - c(\tilde{g}(\theta)) - (u(g(\theta), \theta) - c(\tilde{g}(\theta))))\, f(\theta)d\theta. \quad \text{(A23)}$$

We will show that $\Delta(\varepsilon) > 0$ if ε is sufficiently small, because the second term is positive and is of higher order than ε, while the first term may be negative but is at most of order ε.

To establish the claim regarding the first term, pick some $\bar{\varepsilon} > 0$ and define $\omega = \max_{\theta \in [\underline{\theta}/2, \widehat{\theta}(\bar{\varepsilon})], q \in [0, \bar{\varepsilon}]} (\partial G(q, \theta)/\partial q)$. Note that $\omega < \infty$, and by the Weierstrass Theorem, $\forall\, \varepsilon \le \bar{\varepsilon}$, $G(\tilde{q}(\theta), \theta) - G(q(\theta), \theta) \le \omega(\tilde{q}(\theta) - q(\theta)) \le \omega\varepsilon$, and so $\int_0^1 (G(\tilde{q}(\theta), \theta) - G(q(\theta), \theta))\, f(\theta)d\theta \le \omega\varepsilon$.

Now let us focus on the second term. If $\theta \ge \widehat{\theta}(\varepsilon)$, then $q(\theta) = \tilde{q}(\theta)$ and $U(\theta) < \tilde{U}(\theta)$, so $\tilde{r}(\theta) < r(\theta)$. Also, $\tilde{r}(\theta) = \underline{\theta}/2$ $\forall \theta \in [\underline{\theta}/2, \widehat{\theta}(\varepsilon)]$, while $r(\theta) = \theta\ \forall\, \theta \in [\underline{\theta}/2, \underline{\theta}]$ and $r(\theta) > \underline{\theta}\ \forall\, \theta \in (\underline{\theta}, \widehat{\theta}(\varepsilon)]$. Thus, $\tilde{r}(\theta) < r(\theta)$ $\forall\, \theta \in (\underline{\theta}/2, 1]$ and, by Lemma A7, $q^*(\theta) \ge \tilde{g}(\theta) \ge g(\theta)\ \forall\, \theta \in [0, 1]$. Therefore, since $u(q, \theta) - c(q)$ is concave, $u(\tilde{g}(\theta), \theta) - c(\tilde{g}(\theta)) \ge u(g(\theta), \theta) - c(g(\theta))\ \forall\, \theta \in [0, 1]$. By Lemma A4, $g(\theta) = 0\ \forall\, \theta \in [0, \underline{\theta}]$. So,

$$\int_0^1 (u(\tilde{g}(\theta), \theta) - c(\tilde{g}(\theta)) - (u(g(\theta), \theta) - c(g(\theta))))\, f(\theta)d\theta \ge \int_{\underline{\theta}/2}^{\underline{\theta}} (u(\tilde{g}(\theta), \theta) - c(\tilde{g}(\theta)))\, f(\theta)d\theta.$$

Let us show that $\tilde{g}(\theta) = q(\tilde{r}^{-1}(\theta)) < q^*(\theta)\ \forall\, \theta \in [\underline{\theta}/2, \underline{\theta}]$. First, we establish that $\lim_{\varepsilon \to 0} \tilde{r}^{-1}(\underline{\theta}) = \underline{\theta}$.[34] For suppose, to the contrary, that there exists a sequence ε_n, $\lim_{n\to\infty} \varepsilon_n = 0$, and $\eta > 0$ s.t. $\theta_n = \tilde{r}^{-1}(\underline{\theta}, \varepsilon_n)$ (we need to explicitly incorporate the dependence on ε_n in this argument because we are dealing with a particular sequence) and $\lim_{n\to\infty} \theta_n \ge \underline{\theta} + \eta$. Let θ_ℓ denote the limit of a converging subsequence of θ_n. Obviously, $\theta_\ell \ge \underline{\theta} + \eta$. Note that $\tilde{U}(\theta_n, \varepsilon_n) = u(q(\theta_n), \theta_n) - u(q(\theta_n), \underline{\theta})$. Since $\tilde{U}(\theta, \varepsilon_n)$ converges to $U(\theta)$ uniformly as ε_n converges to zero, and $U(\theta)$ is continuous, we have $U(\theta_\ell) = u(q(\theta_\ell), \theta_\ell) - u(q(\theta_\ell), \underline{\theta})$. On the other hand, since $q(\theta)$ is continuous and $q(\underline{\theta}) = 0$, $U(\theta_\ell) = \int_{\underline{\theta}}^{\theta_\ell} u_\theta(q(s), s)ds < u(q(\theta_\ell), \theta_\ell) - u(q(\theta_\ell), \underline{\theta})$. This contradiction implies that $\lim_{\varepsilon\to 0} \tilde{r}^{-1}(\underline{\theta}) \le \underline{\theta}$. But since $\tilde{r}^{-1}(\underline{\theta}) \ge r^{-1}(\underline{\theta}) = \underline{\theta}\ \forall\, \varepsilon > 0$, we conclude that $\lim_{\varepsilon\to 0} \tilde{r}^{-1}(\underline{\theta}) = \underline{\theta}$.

Now fix some $\psi \in (0, q^*(\underline{\theta}/2))$. Since $\lim_{\varepsilon\to 0} \tilde{r}^{-1}(\underline{\theta}) = \underline{\theta}$ and $q(\theta)$ is continuous, there exists $\widehat{\varepsilon} > 0$ s.t. $q(\tilde{r}^{-1}(\underline{\theta})) \le q^*(\underline{\theta}/2) - \psi\ \forall\, \varepsilon < \widehat{\varepsilon}$, and so $\tilde{g}(\theta) = q(\tilde{r}^{-1}(\theta)) \le q^*(\underline{\theta}/2) - \psi\ \forall\, \theta \in [\underline{\theta}/2, \underline{\theta}]$.

Let $\zeta = \min_{\theta \in [\underline{\theta}/2, \underline{\theta}], q \in [0, q^*(\underline{\theta}/2) - \psi]} u_q(q, \theta) - c'(q)$ and $\widehat{f} = \min_{\theta \in [\underline{\theta}/2, \underline{\theta}]} f(\theta)$. Note that $\zeta > 0$ and $\widehat{f} > 0$. Then $\int_{\underline{\theta}/2}^{\underline{\theta}} (u(\tilde{g}(\theta), \theta) - c(\tilde{g}(\theta)))\, f(\theta)d\theta \ge \zeta \widehat{f} \int_{\underline{\theta}/2}^{\underline{\theta}} \tilde{g}(\theta)d\theta$.

Next we establish a lower bound on $\tilde{g}(\theta)$ for $\theta \in [\underline{\theta}/2, \overline{\theta}]$. Let $m = \min_{\theta \in [\underline{\theta}/2, 1], q \in [\underline{\theta}/2, q^*(\underline{\theta}/2)]} u_{q\theta}(q, \theta)$ and $M = \max_{\theta \in [0,1], q \in [\underline{\theta}/2, q^*(\underline{\theta}/2)]} u_{q\theta}(q, \theta)$. Our assumptions on $u(q, \theta)$ imply that $0 < m \le M < \infty$. Then

$$\begin{aligned} \tilde{U}(\tilde{r}^{-1}(\theta)) = u(\tilde{g}(\theta), \tilde{r}^{-1}(\theta)) - u(\tilde{g}(\theta), \theta) = \int_\theta^{\tilde{r}^{-1}(\theta)} u_\theta(\tilde{g}(\theta), s)ds = \int_\theta^{\tilde{r}^{-1}(\theta)} \int_0^{\tilde{g}(\theta)} u_{\theta q}(q, s)dqds \\ \le M\tilde{g}(\theta)(\tilde{r}^{-1}(\theta) - \theta). \end{aligned} \quad \text{(A24)}$$

On the other hand, since $\tilde{r}^{-1}(\theta) \ge \underline{\theta}$,

$$\tilde{U}(\tilde{r}^{-1}(\theta)) \ge \tilde{U}(\underline{\theta}) = \int_0^{\underline{\theta}} u_\theta(\varepsilon, s)ds = \int_0^{\underline{\theta}} \int_0^{\varepsilon} u_{\theta q}(q, s)dsdq \ge m\varepsilon\underline{\theta}. \quad \text{(A25)}$$

Combining (A24) and (A25), we obtain that $\tilde{g}(\theta) \ge m\varepsilon\underline{\theta}/M(\tilde{r}^{-1}(\theta) - \theta)$. Therefore,

$$\int_{\underline{\theta}/2}^{\underline{\theta}} (u(\tilde{g}(\theta), \theta) - c(\tilde{g}(\theta)))\, f(\theta)d\theta \ge \frac{\zeta \widehat{f} m\varepsilon\underline{\theta}}{M} \int_{\underline{\theta}/2}^{\underline{\theta}} \frac{d\theta}{\tilde{r}^{-1}(\theta) - \theta}.$$

To complete the proof, we will show that $\int_{\underline{\theta}/2}^{\underline{\theta}} d\theta/(\tilde{r}^{-1}(\theta) - \theta)$ increases to ∞ as ε converges to zero. Fix some $\rho \in [0, \underline{\theta}/2]$. Since $\lim_{\varepsilon\to 0} \tilde{r}^{-1}(\theta) = \underline{\theta}\ \forall\, \theta \in [\underline{\theta}/2, \underline{\theta}]$, by Lebesgue's dominated convergence theorem,

$$\lim_{\varepsilon\to 0} \int_{\underline{\theta}/2}^{\underline{\theta}-\rho} \frac{d\theta}{\tilde{r}^{-1}(\theta) - \theta} = \int_{\underline{\theta}/2}^{\underline{\theta}-\rho} \frac{d\theta}{\underline{\theta} - \theta} = \log(\underline{\theta}/2) - \log(\rho).$$

Note that $\lim_{\rho\to 0} \log(\rho) = -\infty$, which proves the desired result. *Q.E.D.*

[34] Note that, as shown in Lemma A6, $q(\tilde{r}^{-1}(\theta))$ is well defined, but the preimage $\tilde{r}^{-1}(\theta)$ may be an interval. However, all the arguments in this article apply to every element in the preimage $\tilde{r}^{-1}(\theta)$, i.e., each θ' s.t. $\tilde{r}(\theta') = \theta$. In this case, we can set $\tilde{r}^{-1}(\theta) = \max\{\theta' | \tilde{r}(\theta') = \theta\}$.

Proof of Theorem 5. The optimal menu of quantity/transfer pairs $(q(\theta), t^s(\theta))$ offered to the strategic consumers after they announce the lowest valuation can be implemented via a tariff $T^s(q)$ s.t. $T^s(q) = t^s(\theta^s(q)) = u(\theta^s(q), q) - \int_0^{\theta^s(q)} u_\theta(z, q(z))dz = u(\theta^s(q), q) - \int_0^q u_\theta(\theta^s(x), x)[d\theta^s(x)/dx]dx = \int_0^q u_q(\theta^s(x), x)dx$, where $\theta^s(q)$ is the valuation of the strategic type who consumes quantity q. $\theta^s(q)$ is the inverse of $q(\theta)$. Note that the third equality follows by a change of variables, and the fourth equality holds because $u(\theta^s(0), 0) = 0$.

Differentiating, we obtain $(T^s(q)/q)' = [qu_q(q, \theta^s(q)) - \int_0^q u_q(x, \theta^s(x))dx]/q^2$. Since $u_q(q, \theta^s(q)) > 0$ for $q \in (0, q^*(1))$ and $u_q(0, \theta^s(0)) = u_q(0, 0) = 0$, it must be that $u_q(q, \theta^s(q))$ is strictly increasing on some interval $[0, \widehat{q}]$ where $\widehat{q} > 0$, and so $qu_q(q, \theta^s(q)) > \int_0^q u_q(x, \theta^s(x))dx$ for all $q \in (0, \widehat{q}]$, i.e., the tariff exhibits quantity premia on $(0, \widehat{q}]$.

On the other hand, since by Lemma 1 $q(1) = q^*(1)$, and by Lemmas A5 and A9 $q(\theta) \leq q^*(\theta)$ for all $\theta \in [0, 1)$ with strict inequality on $(\theta_h, 1]$ for some $\theta_h < 1$, we have $u_q(q(1), 1) = u_q(q^*(1), 1) = c$ and $u_q(q, \theta^s(q)) \geq u_q(q^*(\theta^s(q)), \theta^s(q)) = c$ for $q \in [0, q(1))$ with strict inequality on $(\theta_h, 1]$. So, $d(T^s(q)/q)/dq|_{q=1} < 0$. By continuity, the same must be true for $q \in (q^d, q(1)]$ for some $q^d < q(1)$.

Now consider the linear quadratic case. We have $T^s(q) = \theta^s(q)q - q^2/2 - \int_0^{\theta^s(q)} q(x)dx$, with $\theta^s(q)$ given by (13) for $q \in [0, \bar{q}_\alpha]$ and by $(q+1)/2$ for $q \in [\bar{q}_\alpha, 1]$, $\bar{q}_\alpha = (\sqrt{1+2\alpha}+3)/3(\sqrt{1+2\alpha}+1)$, and $\bar{\theta}_\alpha = \theta^s(\bar{q}_\alpha) = 2/3 + 1/(3(\sqrt{1+2\alpha}+1))$. Then,

$$\left(\frac{T^s(q)}{q}\right)' = \frac{T^{s'}(q)q - T^s(q)}{q^2} = -\frac{1}{2} + \frac{\int_0^{\theta^s(q)} q(x)dx}{q^2} = -\frac{1}{2} + \frac{\int_0^q z\frac{d\theta^s(z)}{dz}dz}{q^2}. \tag{A26}$$

First, let us show that there exists $\widehat{q}_\alpha \in (0, 1)$ such that $(T^s(q)/q)' > 0$ $((T^s(q)/q)' < 0)$ if $q < \widehat{q}_\alpha$ $(q > \widehat{q}_\alpha)$. As a first step, we will establish that if $(T^s(q_0)/q_0)' \leq 0$, then $(T^s(q_1)/q_1)' < 0$ for all $q_1 > q_0$. Note that $\theta^{s''}(q) = 0$ for $q \geq \bar{q}_\alpha$, and direct differentiation of (13) confirms that $\theta^{s''}(q) < 0$ for $q < \bar{q}_\alpha$. Combining these observations with (A26), we conclude that $(T^s(q_0)/q_0)' \leq 0$ implies that $\theta^{s'}(q_0) < 1$. Since $\theta^{s''}(\cdot) \leq 0$, we also get $\theta^{s'}(q_1) < 1$ for all $q_1 > q_0$. By (A26), it must be the case that $T^{s'}(q_0)q_0 - T^s(q_0) \leq 0$. In turn, $(T^{s'}(q)q - T^s(q))' = T^{s''}(q)q = (\theta^{s'}(q) - 1)q$, which is negative for all $q \geq q_0$. Therefore, $T^{s'}(q_1)q_1 - T^s(q_1) < 0$ for all $q_1 > q_0$, and so by (A26), $(T^s(q_1)/q_1)' < 0$.

Further, since we have shown that $d(T^s(q)/q)/dq|_{q=1} < 0$, it follows that $\widehat{q}_\alpha < 1$. To characterize $\widehat{q}_\alpha$, first use (13) in (A26) to obtain that for $q \in [0, \bar{q}]$,

$$\left(\frac{T^s(q)}{q}\right)' = \frac{1-\alpha}{4-\alpha} + \frac{\sqrt{1+2\alpha}-1}{2}\frac{3^{(\sqrt{1+2\alpha}-1)/2}}{(4-\alpha)}\left(\frac{\sqrt{1+2\alpha}+1}{\sqrt{1+2\alpha}+3}\right)^{(\sqrt{1+2\alpha}-3)/2} q(\theta)^{(\sqrt{1+2\alpha}-3)/2} - \frac{1}{2}. \tag{A27}$$

Consider $\alpha < 4$. Equating (A27) to zero, we can compute $\widehat{q}_\alpha$:

$$\widehat{q}_\alpha = \frac{\sqrt{1+2\alpha}+3}{(\sqrt{1+2\alpha}+1)3}\left(\frac{3(\sqrt{1+2\alpha}-1)}{2+\alpha}\right)^{2/(3-\sqrt{1+2\alpha})}. \tag{A28}$$

Note that $\widehat{q}_\alpha < \bar{q}_\alpha$ for $\alpha < 4$, so (A27) indeed applies. Also, observe that $\widehat{q}_\alpha$ converges to zero as α becomes small.

Finally, let us show that $\widehat{q}_\alpha$ is increasing in α. For $\alpha < 4$, direct differentiation of (A28) establishes that $d\widehat{q}_\alpha/d\alpha > 0$. Further, $\widehat{q}_\alpha < (\sqrt{1+2\alpha}+3)/((\sqrt{1+2\alpha}+1)3) < 1/2$ for $\alpha < 4$. On the other hand, $\theta^s(q) = (3/2 - \log(2q))q$ when $\alpha = 4$ (see footnote 29), which can be used to compute $\widehat{q}_4 = 1/2$.

For $\alpha > 4$, $(T(q)/q)' > 0$ for all $q \in [0, (\sqrt{1+2\alpha}+3)/((\sqrt{1+2\alpha}+1)3)]$, and so $\widehat{q}_\alpha$ belongs to the interval $((\sqrt{1+2\alpha}+3)/(\sqrt{1+2\alpha}+1)3, 1)$ on which $\theta^s(q) = (1+q)/2$ for all $\alpha' \geq \alpha$. By Corollary 1, $U(\theta, \alpha') > U(\theta, \alpha)$ if $\alpha' > \alpha$ and $U(\theta, \alpha) = \int_0^\theta q(x;\alpha)dx$ where the dependence of $q(\cdot)$ on α is made explicit. So for $q \in ((\sqrt{1+2\alpha}+3)/((\sqrt{1+2\alpha}+1)3), 1)$, $(T^s(q)/q)' = -1/2 + [\int_0^{\theta^s(q)} q(x;\alpha)dx]/q^2 = -1/2 + U((1+q)/2;\alpha)/q^2$, which is increasing in α. *Q.E.D.*

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